

**APPENDIX for:  
Daughety and Reinganum,  
“Speaking Up: A Model of Judicial Dissent and Discretionary Review”**

**Derivation of Posterior Estimator of  $p_{SC}$**

The joint density on  $(z_M, z_m, z_{SC})$  is denoted  $h(z_M, z_m, z_{SC})$ , which is taken to be continuous and have positive support on its entire domain. There are two possible reports (opinions,  $s_m$ ) from  $m$ , namely  $z_m$ , a reasoned opinion, and  $\emptyset$ , a non-reasoned opinion, so beliefs  $b(z_M, s_m)$  are either (respectively)  $z_m$  or  $b(z_M, \emptyset)$ . Thus, the estimator of  $p_{SC} = p(z_{SC})$  used by each justice (and by judge  $m$  when forecasting the behavior of every justice) for these two reports is either (respectively)  $\rho(z_M, z_m)$  or  $\rho(z_M, b(z_M, \emptyset))$ . We provide the general form for these estimators below.

For notational convenience we derive the following conditional densities (where  $I \equiv [0,1]$  and  $J \equiv b(z_M, \emptyset)$ ).

$$\begin{aligned} g(z_{SC} | z_M, z_m) &\equiv h(z_M, z_m, z_{SC}) \left[ \int_I h(z_M, z_m, u) du \right]^{-1}, \\ f(z_m | z_M) &\equiv \int_I h(z_M, z_m, r) \left[ \int_I \int_I h(z_M, u, z) du dz \right]^{-1} dr. \end{aligned}$$

Thus,  $g$  provides the density of  $z_{SC}$  (given that the case is heard), conditional on the values of  $z_M$  and  $z_m$ , while  $f$  provides the conditional density of  $z_m$  given the value of  $z_M$ .

Using these we find that:

$$\rho(z_M, z_m) = \int_I p(z) g(z | z_M, z_m) dz,$$

and

$$\rho(z_M, b(z_M, \emptyset)) = \int_J \int_I p(z) g(z | z_M, r) f(r | z_M) \left[ \int_J f(s | z_M) ds \right]^{-1} dr dz.$$

Thus,  $\rho$  is the expected value of  $p_{SC}$  given  $M$ 's opinion ( $z_M$ ) and either the reported value  $z_m$  (if  $m$  wrote a reasoned opinion) or the beliefs  $b(z_M, \emptyset)$  about  $z_m$  if  $m$  provided only a non-reasoned opinion.

**Multiple Sympathetic and Persuadable Justices**

The arguments in the text concerning the case of one sympathetic and persuadable justice generalize straightforwardly to the case of one-to-three sympathetic and persuadable justices (indexed by  $i$ ), again assuming that any remaining justices are predisposed to deny cert. Let  $x^S \equiv \min_i \{x_i\}$  and define a Sympathetic Push Equilibrium as in the text but with  $z_m \in [\max\{x_m, x^S\}, 1]$ .

Proposition 1'. There are only two possible types of pure-strategy equilibrium for the case of one-to-three sympathetic and persuadable justices, assuming that any remaining justices are predisposed to deny cert; at least one pure-strategy equilibrium exists.

- (a) If  $\max_i \{E\{V_i(z_M, z_m) | z_m \in [0, \max\{x_m, x^S\}]\}\} > 0$ , then only a Pull Equilibrium exists.
- (b) If  $\max_i \{E\{V_i(z_M, z_m) | z_m \in [0, 1]\}\} < 0$ , then only a Sympathetic Push Equilibrium exists.
- (c) If  $\max_i \{E\{V_i(z_M, z_m) | z_m \in [0, \max\{x_m, x^S\}]\}\} \leq 0$  and  $\max_i \{E\{V_i(z_M, z_m) | z_m \in [0, 1]\}\} \geq 0$ , then both pull and push equilibria exist.

Proof of Proposition 1'. To see that there cannot be any other type of (pure-strategy) equilibrium, suppose that judge  $m$  writes a non-reasoned dissenting opinion for the set of  $z_m$ -values  $[0, \max\{x_m, x^S\})$  plus some additional subset of values  $b \subset [\max\{x_m, x^S\}, 1]$  (excluding  $b =$

$\{\max\{x_m, x^S\}\}$ ; see footnote 28), and writes a reasoned opinion for the remaining values of  $z_m$ . If all justices would vote to deny cert based on the belief that a non-reasoned dissenting opinion came from the set  $[0, \max\{x_m, x^S\}) \cup b$ , then judge  $m$  has an incentive to defect from writing a non-reasoned opinion to writing a reasoned opinion for some  $z_m \in b$ , since this will provoke cert (since  $\max_i\{V_i(z_M, z_m)\} > 0$  for  $z_m \in b$ ). On the other hand, if some justice would vote to grant cert based on the belief that a non-reasoned opinion came from the set  $[0, \max\{x_m, x^S\}) \cup b$ , then judge  $m$  has an incentive to defect from writing a reasoned opinion to writing a non-reasoned opinion for those values of  $z_m \notin [0, \max\{x_m, x^S\}) \cup b$ . Thus, no such strategies on the part of judge  $m$  can be part of a (pure-strategy) equilibrium.

To see that an equilibrium always exists, note that  $E\{V_i(z_M, z_m) \mid z_m \in [0, x]\}$  is an increasing function of  $x$ , since  $V_i(z_M, z_m)$  is increasing in  $z_m$ , and  $z_M$  and  $z_m$  are affiliated (see Milgrom and Weber, 1982, Theorem 5). Thus, when the hypothesis of part (a) holds, then some justice  $i$  will vote to grant cert, with only a non-reasoned dissenting opinion, under the belief that  $z_m \in [0, \max\{x_m, x^S\})$ , thereby upsetting a push equilibrium. On the other hand, since  $E\{V_i(z_M, z_m) \mid z_m \in [0, x]\}$  is an increasing function of  $x$ , this same justice will have  $E\{V_i(z_M, z_m) \mid z_m \in [0, 1]\} > 0$ , thus supporting a pull equilibrium. When the hypothesis of part (b) holds, then no justice will vote to grant cert under the belief that  $z_m \in [0, 1]$ , upsetting a pull equilibrium. However, since  $E\{V_i(z_M, z_m) \mid z_m \in [0, x]\}$  is an increasing function of  $x$ , no justice will vote to grant cert under the belief that  $z_m \in [0, \max\{x_m, x^S\})$  either, thus supporting a push equilibrium.

Proof of part (a). If judge  $m$  uses a strategy of promoting a case for review if and only if  $z_m \in [\max\{x_m, x^S\}, 1]$ , then upon observing only a non-reasoned dissenting opinion, the justices infer that  $z_m \in [0, \max\{x_m, x^S\})$  and calculate  $V_i(z_M, [0, \max\{x_m, x^S\})) = E\{V_i(z_M, z_m) \mid z_m \in [0, \max\{x_m, x^S\})\}$ . If this is positive for at least one justice (as hypothesized in part (a)), then cert will be granted even without a reasoned opinion. But then it will not be optimal for judge  $m$  to promote cases with  $z_m \in [\max\{x_m, x^S\}, 1]$ , upsetting a push equilibrium. However, suppose that judge  $m$  uses a strategy of writing only a non-reasoned dissenting opinion for any  $z_m \in [0, 1]$ . In this case, upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, 1]$ , and calculate  $V_i(z_M, [0, 1])$ . Since  $V_i(z_M, [0, x])$  is increasing in  $x$ , if  $V_i(z_M, [0, \max\{x_m, x^S\})) > 0$  for some justice  $i$ , then  $V_i(z_M, [0, 1]) > 0$  as well, and thus justice  $i$  will vote to grant cert even if judge  $m$  never writes a reasoned opinion. Thus a pull equilibrium exists.

Proof of part (b). If judge  $m$  uses a strategy of writing only a non-reasoned dissenting opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, 1]$ , and calculate  $V_i(z_M, [0, 1])$ . Under the hypothesis of part (b),  $V_i(z_M, [0, 1]) < 0$  for all justices and so none of them will vote to grant cert. But then judge  $m$  will deviate from writing a non-reasoned opinion to writing a reasoned opinion for  $z_m \in [\max\{x_m, x^S\}, 1]$ , since this will provoke cert. Thus a pull equilibrium cannot occur. However, suppose that judge  $m$  uses a strategy of promoting a case if and only if  $z_m \in [\max\{x_m, x^S\}, 1]$ . Then upon observing a non-reasoned dissenting opinion, the justices infer that  $z_m \in [0, \max\{x_m, x^S\})$ , and calculate  $V_i(z_M, [0, \max\{x_m, x^S\}))$ . Since  $V_i(z_M, [0, x])$  is increasing in  $x$ , if  $V_i(z_M, [0, 1]) < 0$  for all  $i$ , then  $V_i(z_M, [0, \max\{x_m, x^S\})) < 0$  for all  $i$  as well, which implies that no justice will vote to grant cert without a reasoned dissenting opinion. This supports a push equilibrium.

Proof of part (c). If judge  $m$  uses a strategy of writing only a non-reasoned opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned dissenting opinion, the justices infer that  $z_m \in [0, 1]$ , and calculate  $V_i(z_M, [0, 1])$ . Under the hypothesis of part (c),  $V_i(z_M, [0, 1]) \geq 0$  for at least one justice  $i$ , so this justice will vote to grant cert even without a reasoned opinion. In this case, it will be optimal for judge  $m$  to write only a non-reasoned dissenting opinion for any  $z_m \in [0, 1]$ . Thus a pull equilibrium exists. If judge  $m$  uses a strategy of writing a reasoned dissenting opinion if and only if  $z_m \in [\max\{x_m, x^S\}, 1]$ , then upon observing a non-reasoned dissenting opinion, the justices infer that  $z_m \in [0, \max\{x_m, x^S\})$ , and calculate  $V_i(z_M, [0, \max\{x_m, x^S\}))$ . Under the hypothesis of part (c), this expression is non-positive for all  $i$ , so justice  $i$  will not vote to grant cert without a reasoned opinion. Judge  $m$  will be willing to promote a case for  $z_m \in [\max\{x_m, x^S\}, 1]$  if doing so will provoke cert, which is the case. Thus a push equilibrium exists. QED.

### One Unsympathetic and Persuadable Justice

Proof of Proposition 2. Suppose that  $[x_m, x_i]$  is non-empty; if this set is empty, then  $V_i(z_M, [0, x_m) \cup (x_i, 1]) = V_i(z_M, [0, 1])$  and the proof is trivial. To see that there cannot be any other type of (pure-strategy) equilibrium, suppose that judge  $m$  provides only a non-reasoned dissenting opinion for the set of  $z_m$ -values  $[0, x_m) \cup (x_i, 1]$  plus some additional subset of values  $b \subset [x_m, x_i]$  (excluding  $b = \{x_m\}$  and  $b = \{x_i\}$ ; see footnote 29), and provides a reasoned dissenting opinion for the remaining values of  $z_m$ . If justice  $i$  would vote to deny cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x_i, 1] \cup b$ , then judge  $m$  has an incentive to defect from a non-reasoned opinion to a reasoned opinion for some  $z_m \in b$ , since this will provoke cert (since  $V_i(z_M, z_m) > 0$  for  $z_m \in b$ ). On the other hand, if justice  $i$  would vote to grant cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x_i, 1] \cup b$ , then judge  $m$  has an incentive to defect from a reasoned to a non-reasoned opinion for those values of  $z_m \notin [0, x_m) \cup (x_i, 1] \cup b$ . Thus, no such strategies on the part of judge  $m$  can be part of a (pure-strategy) equilibrium.

The reason we are able to establish existence in this case is that  $E\{V_i(z_M, z_m) \mid z_m \in [0, 1]\} = E\{V_i(z_M, z_m) \mid z_m \in [0, x_m) \cup (x_i, 1]\} \Pr\{z_m \in [0, x_m) \cup (x_i, 1]\} + E\{V_i(z_M, z_m) \mid z_m \in [x_m, x_i]\} \Pr\{z_m \in [x_m, x_i]\}$ . Notice that the expression  $E\{V_i(z_M, z_m) \mid z_m \in [x_m, x_i]\} \geq 0$ , when  $[x_m, x_i]$  is non-empty, since  $V_i(z_M, z_m) \geq 0$  for  $z_m \in [x_m, x_i]$ , as can be seen in Figure 3(a). Thus, if  $E\{V_i(z_M, z_m) \mid z_m \in [0, x_m) \cup (x_i, 1]\} > 0$  (upsetting a push equilibrium), then  $E\{V_i(z_M, z_m) \mid z_m \in [0, 1]\} > 0$  (supporting a pull equilibrium). Similarly, if  $E\{V_i(z_M, z_m) \mid z_m \in [0, 1]\} < 0$  (upsetting a pull equilibrium), then  $E\{V_i(z_M, z_m) \mid z_m \in [0, x_m) \cup (x_i, 1]\} < 0$  (supporting a push equilibrium). Thus, if justice  $i$  upsets one type of equilibrium, she guarantees that the other exists. If  $[x_m, x_i]$  is empty then  $\Pr\{z_m \in [x_m, x_i]\} = 0$  and the same argument holds.

Proof of part (a). If judge  $m$  uses a strategy of promoting a case if and only if  $z_m \in [x_m, x_i]$ , then upon observing a non-reasoned opinion, justice  $i$  infers that  $z_m \in [0, x_m) \cup (x_i, 1]$  and calculates  $V_i(z_M, [0, x_m) \cup (x_i, 1]) = E\{V_i(z_M, z_m) \mid z_m \in [0, x_m) \cup (x_i, 1]\}$ . If this is positive for justice  $i$  (as hypothesized in part (a)), then cert will be granted even without a reasoned opinion. But then it will not be optimal for judge  $m$  to promote a case for  $z_m \in [x_m, x_i]$ , upsetting a push equilibrium. However, suppose that judge  $m$  uses a strategy of writing only a non-reasoned dissenting opinion for any  $z_m \in [0, 1]$ . In this case, upon observing a non-reasoned opinion, justice  $i$  infers that  $z_m \in [0, 1]$ , and calculates  $V_i(z_M, [0, 1])$ . Notice that:

$$\begin{aligned}
V_i(z_M, [0, 1]) &= V_i(z_M, [0, x_m] \cup (x_i, 1))\Pr\{z_m \in [0, x_m] \cup (x_i, 1)\} \\
&\quad + V_i(z_M, [x_m, x_i])\Pr\{z_m \in [x_m, x_i]\}. \tag{A.1}
\end{aligned}$$

The expression  $V_i(z_M, [x_m, x_i]) = E\{V_i(z_M, z_m) \mid z_m \in [x_m, x_i]\} \geq 0$  since  $V_i(z_M, z_m) \geq 0$  for  $z_m \in [x_m, x_i]$ , as can be seen in Figure 3(a). Thus, if  $V_i(z_M, [0, x_m] \cup (x_i, 1)) > 0$ , then so is  $V_i(z_M, [0, 1]) > 0$ , and thus a pull equilibrium exists.

Proof of part (b). If judge m uses a strategy of providing only a non-reasoned opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned opinion, justice i infers that  $z_m \in [0, 1]$ , and calculates  $V_i(z_M, [0, 1])$ . Under the hypothesis of part (b),  $V_i(z_M, [0, 1]) < 0$  and so cert is denied without a reasoned opinion. But then judge m will deviate from a non-reasoned to a reasoned dissenting opinion for  $z_m \in [x_m, x_i]$ , since this will provoke cert. Thus a pull equilibrium cannot occur. However, suppose that judge m uses a strategy of promoting a case if and only if  $z_m \in [x_m, x_i]$ . Then upon observing a non-reasoned opinion, justice i infers that  $z_m \in [0, x_m] \cup (x_i, 1)$ , and calculates  $V_i(z_M, [0, x_m] \cup (x_i, 1))$ . Referring to equation (A.1), we see that if  $V_i(z_M, [0, 1]) < 0$ , then so is  $V_i(z_M, [0, x_m] \cup (x_i, 1)) < 0$ , and thus a push equilibrium exists.

Proof of part (c). If judge m uses a strategy of providing only a non-reasoned opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned opinion, justice i infers that  $z_m \in [0, 1]$ , and calculates  $V_i(z_M, [0, 1])$ . Under the hypothesis of part (c),  $V_i(z_M, [0, 1]) \geq 0$ , so justice i will vote to grant cert even without a reasoned opinion. In this case, it will be optimal for judge m to provide only a non-reasoned opinion for any  $z_m \in [0, 1]$ . Thus a pull equilibrium exists. If judge m uses a strategy of promoting a case if and only if  $z_m \in [x_m, x_i]$ , then upon observing a non-reasoned opinion, justice i infers that  $z_m \in [0, x_m] \cup (x_i, 1)$ , and calculates  $V_i(z_M, [0, x_m] \cup (x_i, 1))$ . Under the hypothesis of part (c), this expression is non-positive, so justice i will vote to deny cert without a reasoned opinion. Judge m will be willing to write for  $z_m \in [x_m, x_i]$  if doing so will provoke cert, which is the case. Thus a push equilibrium exists. QED.

### Multiple Unsympathetic and Persuadable Justices

The arguments in the text characterizing reporting intervals generalize straightforwardly to the case of one-to-three unsympathetic and persuadable justices (indexed by i), again assuming that any remaining justices are predisposed to deny cert. Let  $x^U \equiv \max_i \{x_i\}$  and define an Unsympathetic Push Equilibrium as in the text but with  $z_m \in [x_m, x^U]$ .

Proposition 2'. There are only two possible types of (pure-strategy) equilibrium for the case of one-to-three unsympathetic and persuadable justices, assuming that any remaining justices are predisposed to deny cert.

(a) If  $\max_i \{V_i(z_M, [0, x_m] \cup (x^U, 1))\} = \max_i \{E\{V_i(z_M, z_m) \mid z_m \in [0, x_m] \cup (x^U, 1)\}\} \leq 0$ , then an Unsympathetic Push Equilibrium exists.

(b) If  $\max_i \{V_i(z_M, [0, 1])\} = \max_i \{E\{V_i(z_M, z_m) \mid z_m \in [0, 1]\}\} \geq 0$ , then a Pull Equilibrium exists.

Proof of Proposition 2'. To see that there cannot be any other type of (pure-strategy) equilibrium, suppose that judge m provides only a non-reasoned opinion for a set of  $z_m$ -values  $[0, x_m] \cup (x^U, 1]$

plus some additional subset of values  $b \subset [x_m, x^U]$  (excluding  $b = \{x_m\}$  and  $b = \{x^U\}$ ; see footnote 29), and promotes the case for the remaining values of  $z_m$ . If all justices would vote to deny cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x^U, 1] \cup b$ , then judge  $m$  has an incentive to defect from a non-reasoned to a reasoned opinion for some  $z_m \in b$ , since this will provoke cert (since  $\max_i \{V_i(z_M, z_m)\} > 0$  for  $z_m \in b$ ). On the other hand, if some justice would vote to grant cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x^U, 1] \cup b$ , then judge  $m$  has an incentive to defect from a reasoned to a non-reasoned opinion for those values of  $z_m \notin [0, x_m) \cup (x^U, 1] \cup b$ . Thus, no such strategies on the part of judge  $m$  can be part of a (pure-strategy) equilibrium.

**Proof of part (a).** If judge  $m$  uses a strategy of promoting a case if and only if  $z_m \in [x_m, x^U]$ , then upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, x_m) \cup (x^U, 1]$  and calculate  $V_i(z_M, [0, x_m) \cup (x^U, 1]) = E\{V_i(z_M, z_m) \mid z_m \in [0, x_m) \cup (x^U, 1]\}$ . If this is non-positive for all justices (as hypothesized in part (a)), then cert will not be granted without a reasoned opinion. But then it will be optimal for judge  $m$  to promote a case if and only if  $z_m \in [x_m, x^U]$ , which provokes cert. Thus a push equilibrium exists under the hypothesis of part (a).

**Proof of part (b).** If judge  $m$  uses a strategy of providing only a non-reasoned opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, 1]$ , and calculate  $V_i(z_M, [0, 1])$ . Under the hypothesis of part (b),  $\max_i \{V_i(z_M, [0, 1])\} \geq 0$  so at least one justice will vote to grant cert with only a non-reasoned opinion. But then it will be optimal for judge  $m$  to write only a non-reasoned opinion for any  $z_m \in [0, 1]$ , so a pull equilibrium exists under the hypothesis of part (b). QED

**Comment.** Proposition 2' is weaker than Proposition 2 because the argument involving equation (A.1) does not extend to multiple unsympathetic justices, since it need not be true that  $V_i(z_M, [x_m, x^U]) \geq 0$  for all  $i$ . Thus different justices can upset a pull versus a push equilibrium. The hypothesis that  $V_i(z_M, [0, x_m) \cup (x_i, 1]) > 0$  for some  $i$  (upsetting a push equilibrium) does not imply that  $V_i(z_M, [0, 1]) \geq 0$ . Similarly, the hypothesis that  $V_i(z_M, [0, 1]) < 0$  for all  $i$  (upsetting a pull equilibrium) does not imply that  $V_i(z_M, [0, x_m) \cup (x^U, 1]) \leq 0$  for all  $i$ . Consequently, it is possible for neither of the hypotheses to hold, in which case there is no pure-strategy equilibrium. The nature of the mixing needed to support a mixed-strategy equilibrium is that one or more justices must randomize after receiving a non-reasoned dissenting opinion.

### Push Equilibrium with Conflicting Sympathies

We consider the case of conflicting sympathies on the part of the justices. Assume that one justice  $i$  is sympathetic and persuadable, while another justice  $j$  is unsympathetic and persuadable, and assume that the third justice is predisposed to deny cert. Then  $V_i(z_M, z_m)$  is increasing in  $z_m$  with critical value  $x_i$  and  $V_j(z_M, z_m)$  is decreasing in  $z_m$  with critical value  $x_j$ . The form of a candidate for a pure strategy push equilibrium can be obtained through the following argument. Judge  $m$  will never write a reasoned dissent for  $z_m \in [0, x_m)$ , even if doing so would provoke cert. However, judge  $m$  will be able to persuade the unsympathetic justice  $j$  to vote to grant cert by writing a reasoned dissent for  $z_m \in [x_m, x_j]$ , where we take the interval  $[x_m, x_j]$  to be empty if  $x_m > x_j$ . In addition, judge  $m$  will be able to persuade the sympathetic justice  $i$  to vote to grant cert by writing a reasoned

dissent for  $z_m \in [x_i, 1]$ . Combining these three observations implies that the region in which judge  $m$  will find it optimal to write a reasoned dissent in order to provoke cert is given by  $z_m \in [x_m, x_j] \cup [\max\{x_m, x_i\}, 1]$ .

The case of one sympathetic and persuadable justice  $i$  and one unsympathetic and persuadable justice  $j$  is depicted in Figure A1 below for  $x_i > x_j$  (the case of  $x_i \leq x_j$ , which leads to a mixed strategy equilibrium, is discussed below). There are multiple possible locations for  $x_m$  relative to  $x_i$  and  $x_j$ , but the analysis is straightforward for all possible locations of  $x_m$ ; Figure A1 illustrates the case of  $x_m < x_j$ . The values of  $z_m$  for which judge  $m$  writes a reasoned dissent in equilibrium are darkened for emphasis.

Note that, once again, there are “strange bedfellows” in this equilibrium. The intensity of opposition of justice  $i$  and justice  $j$  has led to a “hole” in the usually-connected interval representing the values of  $z_m$  for which judge  $m$  would write a reasoned dissent. Thus, in this example, judge  $m$

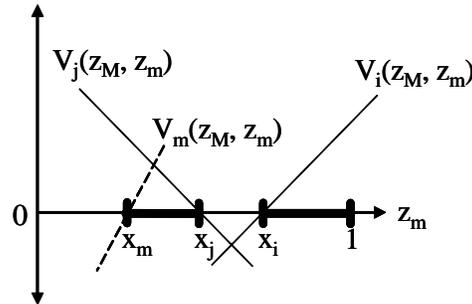


Figure A1:  $V_i$ ,  $V_j$ ,  $V_m$  and Mixed-Sympathies Push Equilibria

is writing either to persuade justice  $i$  or justice  $j$ , but not both.

**Definition. Mixed-Sympathies Push Equilibrium:** All justices vote to deny cert without a reasoned dissent. Judge  $m$  writes a reasoned dissent if and only if  $z_m \in [x_m, x_j] \cup [\max\{x_m, x_i\}, 1]$ ; at least one justice votes to grant cert upon receipt of such an opinion.

The following proposition summarizes conditions under which each type of equilibrium can exist; its proof is subsumed by that provided for Proposition A1' below.

**Proposition A1.** There are only two possible types of pure-strategy equilibrium for a collection of persuadable justices, one of whom is sympathetic and one of whom is unsympathetic (with the third justice predisposed to deny cert).

- (a) If  $\max_k \{E\{V_k(z_M, z_m) \mid z_m \in [0, x_m] \cup (x_j, x_i)\}\} \leq 0$ , then a Mixed-Sympathies Push Equilibrium exists.
- (b) If  $\max_k \{E\{V_k(z_M, z_m) \mid z_m \in [0, 1]\}\} \geq 0$ , then a Pull Equilibrium exists.

Both hypotheses above could hold simultaneously, supporting both types of equilibrium.

However, we cannot rule out the possibility that neither hypothesis holds, in which case there is no pure-strategy equilibrium; the nature of the mixing needed to support a mixed-strategy equilibrium is that one or more justices must randomize after receiving only a non-reasoned dissent.

### Multiple Justices with Mixed Sympathies

The arguments in the text characterizing reporting intervals generalize straightforwardly to the remaining cases of (i) one sympathetic justice and two unsympathetic justices; and (ii) two sympathetic justices and one unsympathetic justice. Let  $x^S = \min_i \{x_i\}$ , where  $i$  denotes a sympathetic and persuadable justice, and let  $x^U = \max_j \{x_j\}$ , where  $j$  denotes an unsympathetic and persuadable justice. Define a Mixed-Sympathies Push Equilibrium as above, but with  $z_m \in [x_m, x^U] \cup [\max\{x_m, x^S\}, 1]$ .

Proposition A1'. There are only two possible types of pure-strategy equilibrium for a collection of persuadable justices, some of whom are sympathetic, some of whom are unsympathetic and (at most) one of whom is predisposed to deny cert.

(a) If  $\max_k \{E\{V_k(z_M, z_m) \mid z_m \in [0, x_m) \cup (x^U, x^S)\}\} \leq 0$ , then a Mixed-Sympathies Push Equilibrium exists.

(b) If  $\max_k \{E\{V_k(z_M, z_m) \mid z_m \in [0, 1]\}\} \geq 0$ , then a Pull Equilibrium exists.

Proof of Proposition A1'. To see that there cannot be any other type of (pure-strategy) equilibrium, suppose that judge  $m$  provides only a non-reasoned opinion for a set of  $z_m$ -values  $[0, x_m) \cup (x^U, x^S)$  plus some additional subset of values  $b \subset [x_m, x^U] \cup [\max\{x_m, x^S\}, 1]$  (excluding  $b = \{x_m\}$ ,  $b = \{x^U\}$ , and  $b = \{\max\{x_m, x^S\}\}$ ; see footnotes 28 and 29), and promotes the case for the remaining values of  $z_m$ . If all justices would vote to deny cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x^U, x^S) \cup b$ , then judge  $m$  has an incentive to defect from a non-reasoned to a reasoned opinion for some  $z_m \in b$ , since this will provoke cert (since  $\max_k \{V_k(z_M, z_m)\} > 0$  for  $z_m \in b$ ). On the other hand, if some justice would vote to grant cert based on the belief that a non-reasoned opinion came from the set  $[0, x_m) \cup (x^U, x^S) \cup b$ , then judge  $m$  has an incentive to defect from a reasoned to a non-reasoned opinion for those values of  $z_m \notin [0, x_m) \cup (x^U, x^S) \cup b$ . Thus, no such strategies on the part of judge  $m$  can be part of a (pure-strategy) equilibrium.

Proof of part (a). If judge  $m$  uses a strategy of promoting a case if and only if  $z_m \in [x_m, x^U] \cup [\max\{x_m, x^S\}, 1]$ , then upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, x_m) \cup (x^U, x^S)$  and calculate  $V_k(z_M, [0, x_m) \cup (x^U, x^S)) = E\{V_k(z_M, z_m) \mid z_m \in [0, x_m) \cup (x^U, x^S)\}$ . If this is non-positive for all justices (as hypothesized in part (a)), then cert will not be granted without a reasoned opinion. But then it will be optimal for judge  $m$  to promote the case if and only if  $z_m \in [x_m, x^U] \cup [\max\{x_m, x^S\}, 1]$ , which provokes cert. Thus a push equilibrium exists under the hypothesis of part (a).

Proof of part (b). If judge  $m$  uses a strategy of providing only a non-reasoned opinion for any  $z_m \in [0, 1]$ , then upon observing a non-reasoned opinion, the justices infer that  $z_m \in [0, 1]$ , and calculate  $V_k(z_M, [0, 1])$ . Under the hypothesis of part (b),  $\max_k \{V_k(z_M, [0, 1])\} \geq 0$  so at least one justice will vote to grant cert with only a non-reasoned opinion. But then it will be optimal for judge  $m$  to provide only a non-reasoned opinion for any  $z_m \in [0, 1]$ , so a pull equilibrium exists under the

hypothesis of part (b). QED.

As was observed after Proposition A1, both hypotheses above could hold simultaneously, supporting both types of equilibrium. However, we cannot rule out the possibility that neither of these hypotheses holds, in which case there is no pure-strategy equilibrium; the nature of the mixing needed to support a mixed-strategy equilibrium is that one or more justices must randomize after receiving non-reasoned dissenting opinion. For example, the need for randomization may arise when  $x_m < x_i < x_j$  and there is one sympathetic and persuadable justice and one unsympathetic and persuadable justice. The candidate equilibrium reporting set  $[x_m, x_j] \cup [\max\{x_m, x_i\}, 1]$  reduces to  $[x_m, 1]$ , and thus the non-reporting set is simply  $[0, x_m)$ . However, since  $V^j(z_M, z_m) > 0$  for all  $z_m \in [0, x_m)$ , it follows that  $E\{V_j(z_M, z_m) \mid z_m \in [0, x_m)\} > 0$ , and thus the unsympathetic justice will vote for cert upon receiving only a non-reasoned dissent (upsetting a push equilibrium). If conditions were such that the pull equilibrium also did not exist, one could construct a mixed-strategy push equilibrium, wherein at least one justice would randomize over voting to grant cert when judge m provides only a non-reasoned opinion, and judge m would promote a case for a smaller domain of  $z_m$  values than  $[x_m, 1]$ .

### Derivation of Comparative Statics Effects

**Lemma A1.** If justice i is both sympathetic and persuadable, then  $\gamma_i^P > k_{SC} > \gamma_i^R$ . If justice i is unsympathetic and persuadable, then  $\gamma_i^R > k_{SC} > \gamma_i^P$ .

**Proof of Lemma A1.** We can write  $V_i(z_M, z_m)$  as  $\rho(z_M, z_m)(\gamma_i^P - k_{SC}) + (1 - \rho(z_M, z_m))(\gamma_i^R - k_{SC})$ . Sympathetic means that  $(\gamma_i^P - \gamma_i^R) > 0$  while persuadable means that  $(\gamma_i^P - k_{SC})$  and  $(\gamma_i^R - k_{SC})$  are of opposite sign, so that if justice i is both sympathetic and persuadable, then  $\gamma_i^P > k_{SC} > \gamma_i^R$ . If justice i is unsympathetic then  $(\gamma_i^P - \gamma_i^R) < 0$ , so that being both unsympathetic and persuadable implies that  $\gamma_i^R > k_{SC} > \gamma_i^P$ . QED

**Proof of Proposition 3.** Recall that if justice i is sympathetic and persuadable, then  $(\gamma_i^P - \gamma_i^R) > 0$ . On the other hand, if justice i is unsympathetic and persuadable, then  $(\gamma_i^P - \gamma_i^R) < 0$ .

(a) The critical value  $x_m$  is defined implicitly by the equation  $\rho(z_M, x_m) = [u^{SW} - u^W + k_{AC}]/[u^B - u^W]$ , where  $\rho(z_M, z_m)$  is strictly increasing in both arguments. First note that  $x_m$  is independent of  $u^{SB}$ ,  $k_{SC}$ ,  $\alpha_i^P$  and  $\alpha_i^R$ . Differentiating and collecting terms implies:

- (a.1)  $dx_m/dz_M = -(\partial\rho/\partial z_M)/(\partial\rho/\partial z_m) < 0$ .
- (a.2)  $dx_m/du^B = -\rho/(u^B - u^W)(\partial\rho/\partial z_m) < 0$ .
- (a.3)  $dx_m/du^{SW} = 1/(u^B - u^W)(\partial\rho/\partial z_m) > 0$ .
- (a.4)  $dx_m/du^W = -(1 - \rho)/(u^B - u^W)(\partial\rho/\partial z_m) < 0$ .
- (a.5)  $dx_m/dk_{AC} = 1/(u^B - u^W)(\partial\rho/\partial z_m) > 0$ .

(b) The critical value  $x_i$  is defined implicitly by the equation  $V_i(z_M, x_i) = \rho(z_M, x_i)(\gamma_i^P - \gamma_i^R) + \gamma_i^R - k_{SC} = 0$  or, equivalently,  $V_i(z_M, x_i) = \rho(z_M, x_i)\gamma_i^P + (1 - \rho(z_M, x_i))\gamma_i^R - k_{SC} = 0$ . First note that  $x_i$  is independent of  $k_{AC}$ . Differentiating and collecting terms, taking into account how  $\gamma_i^P$  and  $\gamma_i^R$  depend on  $u^B$ ,  $u^{SB}$ ,  $u^{SW}$ ,  $u^W$ ,  $\alpha_i^P$  and  $\alpha_i^R$ , implies:

- (b.1)  $dx_i/dz_M = -(\partial\rho/\partial z_M)/(\partial\rho/\partial z_m) < 0$ .  
 (b.2)  $dx_i/du^B = -[\rho\alpha_i^P + (1-\rho)\alpha_i^R]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) < 0$ .  
 (b.3)  $dx_i/du^{SB} = [\rho(1-\alpha_i^P) + (1-\rho)\alpha_i^R]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) > 0$ .  
 (b.4)  $dx_i/du^{SW} = [\rho\alpha_i^P + (1-\rho)(1-\alpha_i^R)]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) > 0$ .  
 (b.5)  $dx_i/du^W = -[\rho(1-\alpha_i^P) + (1-\rho)(1-\alpha_i^R)]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) < 0$ .  
 (b.6)  $dx_i/d\alpha_i^P = -\rho[u^B - u^W + (u^{SB} - u^{SW})]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) < 0$ .  
 (b.7)  $dx_i/d\alpha_i^R = -(1-\rho)[u^B - u^W - (u^{SB} - u^{SW})]/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) < 0$ .  
 (b.8)  $dx_i/dk_{SC} = 1/(\gamma_i^P - \gamma_i^R)(\partial\rho/\partial z_m) > 0$ . QED

(c) The critical value  $x_i$  is defined by the same equation, and the same formulae obtain as in part (b) above. However, with the exception of  $dx_i/dz_M$ , the results are of opposite sign since  $(\gamma_i^P - \gamma_i^R) < 0$ . QED

### Derivation of Figure 5

We augment the notation for the function  $V_i(z_M, [0, x])$  to reflect its dependence on  $k_{SC}$ , since  $k_{SC}$  enters additively with a negative sign and enters again through  $x_i(k_{SC})$ :  $V_i(z_M, [0, x]; k_{SC})$ . For  $k_{SC} \in [0, k_{SC2}]$ , the expression of interest is  $V_i(z_M, [0, x_m]; k_{SC})$ ; on the other hand, for  $k_{SC} \in [k_{SC2}, k_{SC3}]$ , the expression of interest is  $V_i(z_M, [0, x_i(k_{SC})]; k_{SC})$ .

Claim A1. a)  $V_i(z_M, [0, x_m]; 0) > 0$ ; and b)  $V_i(z_M, [0, x_m]; k_{SC2}) < 0$ .

Proof of Claim A1 a). Since  $V_i(z_M, 0; 0) = 0$  (this follows from the fact that  $x_i(0) = 0$ ) and  $V_i$  is increasing in  $z_m$ , it follows that  $V_i(z_M, z_m; 0) > 0$  for all  $z_m \in (0, 1]$ . Thus,  $V_i(z_M, [0, x_m]; 0) > 0$ .

Proof of Claim A1 b). Since  $V_i(z_M, x_m; k_{SC2}) = 0$  (this follows from  $x_i(k_{SC2}) = x_m$ ) and  $V_i$  is increasing in  $z_m$ , it follows that  $V_i(z_M, z_m; k_{SC2}) < 0$  for all  $z_m \in [0, x_m)$ . Thus  $V_i(z_M, [0, x_m]; k_{SC2}) < 0$ . QED

Claim A1, combined with the fact that  $V_i(z_M, [0, x_m]; k_{SC})$  is continuous and strictly decreasing in  $k_{SC}$ , implies that there exists a unique value  $k_{SC1} \in (0, k_{SC2})$  such that  $V_i(z_M, [0, x_m]; k_{SC1}) = 0$  (as claimed in the text).

Claim A2.  $V_i(z_M, [0, x_i(k_{SC})]; k_{SC}) < 0$ .

Proof of Claim A2. Since  $V_i(z_M, x_i(k_{SC}); k_{SC}) = 0$  (this follows from the definition of  $x_i$ ) and, since  $V_i$  is increasing in  $z_m$ , it follows that  $V_i(z_M, z_m; k_{SC}) < 0$  for all  $z_m \in [0, x_i(k_{SC}))$ . Thus  $V_i(z_M, [0, x_i(k_{SC})]; k_{SC}) < 0$ . QED

Thus, we have established the following. For  $k_{SC} \in [0, k_{SC1})$ , only a pull equilibrium exists: judge  $m$  never promotes a case because, even absent a reasoned opinion, justice  $i$  will vote for cert based on her beliefs about  $z_m$ . However, as  $k_{SC}$  rises, the function  $V_i(z_M, [0, x_m]; k_{SC})$  shifts down vertically and eventually the regime will transit to one wherein  $x_m$  still exceeds  $x_i$ , but now  $V_i(z_M, [0, x_m]; k_{SC}) \leq 0$ ; this transition occurs at the value  $k_{SC1}$ . For  $k_{SC} \geq k_{SC1}$ , it is optimal for justice  $i$  not to vote for cert unless a reasoned opinion (revealing a sufficiently high value of  $z_m$ ) is provided.

Thus, judge m now promotes the case for  $z_m \in [x_m, 1]$  and justice i votes to grant cert if and only if a reasoned opinion reporting  $z_m \in [x_m, 1]$  is received; now a Sympathetic Push Equilibrium exists. As  $k_{SC}$  rises still further,  $x_i(k_{SC})$  rises until it reaches  $x_m$  (this transition occurs at the value  $k_{SC2}$ ). Finally, for  $k_{SC} \in (k_{SC2}, k_{SC3}]$ , a push equilibrium continues to exist, but now judge m promotes the case for  $z_m \in [x_i(k_{SC}), 1]$  and justice i votes to grant cert if and only if a reasoned opinion reporting  $z_m \in [x_i(k_{SC}), 1]$  is received; this set becomes progressively smaller as  $k_{SC}$  rises, until finally the set becomes empty at  $k_{SC3}$ . Note that, while push and pull equilibria can co-exist for values of  $k_{SC} \in [k_{SC1}, k_{SC3})$ , a pull equilibrium cannot exist when  $k_{SC} = k_{SC3}$  (since  $V_i$  is negative for all  $z_m < 1$ ), and therefore a pull equilibrium ceases to exist at some point within  $[k_{SC1}, k_{SC3})$ .

**Analog of Figure 5 for One Unsympathetic Justice**

Figure A2 provides the diagram analogous to Figure 5 in the text, except now we consider the case wherein there is one unsympathetic and persuadable justice and the rest are predisposed to deny cert. Note that, even if push equilibria exist at moderate levels of  $k_{SC}$ , sufficiently large values of this cost may eliminate equilibria wherein judge m writes a reasoned dissent.

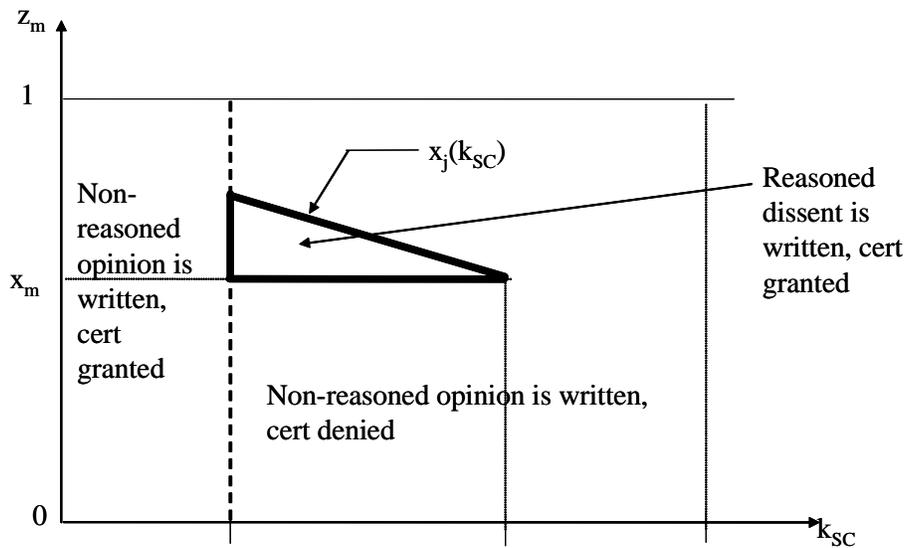


Figure A2: Equilibrium Outcomes as a Function of  $k_{SC}$  for the Unsympathetic Justice Case