# Hidden Talents: Entrepreneurship and Pareto-Improving Private Information


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*Two entrepreneurs, each privately informed about her own talent, simultaneously and noncooperatively choose their efforts in producing a new product. Product quality depends on both entrepreneurs' talents and efforts, but is unobservable by potential buyers prior to purchase; however, buyers can observe the entrepreneurs' individual efforts. Because the entrepreneurs share the payoff, each is tempted to shirk. However, the need to signal quality to potential buyers serves as a credible commitment to provide greater effort. Thus, the "problem" of adverse selection mitigates the problem of moral hazard, so that a new venture can perform better than the corresponding mature market.*


## 1. Introduction

An entrepreneur is, almost by definition, someone whose talent in a new venture is unknown by other market participants; on the other hand, the entrepreneur is likely to have some private information about her own talent in the new venture. For example, she may provide a service, or a new product, whose quality depends on her unobservable talent and her observable effort. Although quality may be unobservable by potential buyers in advance of purchase, the entrepreneur may be able to signal her product's quality to the market through her choice of effort. Such signaling usually involves distortion of the effort level away from


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its complete-information level, and is thus costly relative to a situation of complete information.

In this paper, we model a more complicated interaction between entrepreneurs and potential buyers. In particular, we consider two entrepreneurs who contribute to the completion of a project or the production of a final good; we will use the term "product" to represent the result of the collaboration. The product is produced through the combined talents and efforts of both entrepreneurs. An entrepreneur's productivity is the sum of her talent and her effort and, in our primary model specification, the value of the product is found by multiplying the entrepreneurs' productivities.[1] We assume that effort is observable, whereas talent is unobservable; each entrepreneur is privately informed only about her own talent. Although effort is assumed to be observable, we assume that it is not verifiable (see, e.g., Grossman and Hart, 1986; Hart and Moore, 1990), and hence the entrepreneurs themselves cannot contract over it; rather, we assume that they share equally the proceeds from the sale of the product.

Potential buyers are also assumed to be able to observe the entrepreneurs' respective efforts, but cannot contract over effort, because effort is unverifiable and because we also envision the potential buyers as arriving on the scene after the effort investments have been sunk. Potential buyers cannot observe the entrepreneurs' talents, but can use their observations of effort to draw inferences about talent. Thus, in addition to its direct contribution to product quality, observable effort serves as a signal for unobservable talent. Because the entrepreneurs share the payoff from the product, each is tempted to shirk in providing effort. However, the need for each entrepreneur to signal the quality of the product to potential buyers serves as a credible commitment to provide greater effort. We show how the distortion involved in signaling talent can (in comparison with a complete-information Nash equilibrium) enhance the value of the product to potential buyers, which in turn results in a higher price and creates greater profits for the entrepreneurs who produce it. More precisely, we find that noncooperative simultaneous[2] signaling need not be wasteful, and can actually be welfare enhancing in the strongest sense: there is a significant portion of the parameter space wherein incomplete information is Pareto-improving relative to the complete-information Nash

1. Payoff functions that are multiplicative in type and/or strategy have previously been employed by Chou (2007), Gervais and Goldstein (2007), Kremer (1993), and Winter (2004). In Section 4 we briefly examine an additive-productivity version of our model, as well as a sole-entrepreneur version.

2. We require the entrepreneurs to move simultaneously so that no form of coordination among them (such as "leader–follower" behavior) can influence the results.

equilibrium for *all* possible nondegenerate prior distributions over the private information.[3]

If we view the typical entrepreneurial market as one involving extensive private information about entrepreneurs' talents, and the typical mature market as one involving essentially complete information about talent, then this result indicates that an entrepreneurial market can actually outperform the corresponding mature market, because the adverse selection "problem" caused by private information can actually mitigate the moral hazard problem.

As an example of an application, the entrepreneurs might be members of a startup firm, wherein one is responsible for the design of the product and another is responsible for the design of the marketing campaign. Once each entrepreneur has completed her task, they try to sell their fledgling firm to a larger firm with manufacturing and distribution capability. Because the entrepreneurs are joint owners of the startup, they split the proceeds equally. Although many aspects of a product design are observable, some of a product's attributes can only be learned through experience with production and/or final consumption. Similarly, although many aspects of the marketing plan are observable, others (such as how effectively the entrepreneur responsible for marketing intuits the "pulse" of final consumers) may also be learned by the acquiring firm only through experience. The entrepreneurs use their observable effort choices to influence the potential buyers' beliefs about the value of acquiring the startup.

Another example involves two entrepreneurs who produce innovative complementary goods that are used by consumers or final-product producers in fixed proportions, such as a new game platform and (a collection of) associated games. For simplicity, assume that one unit of each complementary good is needed to construct the final product. Consumers are willing to pay a lump sum for the combination of one unit of each good, but nothing for each separate component alone. Because both goods are critical inputs to the final product, bargaining between the two entrepreneurs splits the proceeds equally. The quality of each good is partially observable and partially unobservable. For instance, whereas the technical specifications and exterior aspects of a game platform and game cartridges are observable, their durability and ultimate entertainment value (respectively) are attributes that can only be learned by the consumer through experience (but might be known privately by the respective entrepreneurs/inventors). The entrepreneurs can use their respective effort choices to influence consumers' beliefs about the value of the final product.

3. Thus, private information is both privately and socially beneficial.

Finally, the entrepreneurs might be established firms collaborating on a new venture through a strategic alliance. According to Deeds and Rothaermel (2003, p. 469), "Firms are motivated to enter into alliances to access complementary assets and knowledge needed for the successful creation and commercialization of a new product." In particular, they consider biotech startups that partner with another firm along the supply chain, such as a pharmaceutical firm. The biotech firm tends to specialize in the "research" aspect, whereas the pharmaceutical firm tends to specialize in the "development" aspect (including manufacturing and distribution). Each partner may have considerable uncertainty at the outset regarding the other partner's "talent" (here represented by the partner's assets and knowledge that are relevant to the new venture) but this uncertainty may be resolved over time. This suggests that biotech strategic alliances may be an area in which younger alliances outperform more mature alliances.

Deeds and Rothaermel (2003) examine the relationship between the age of an alliance and its performance, using data on 115 individual R&D strategic alliances of biotechnology startups. They examine two alternative stories about how alliances evolve. The first story posits that the relationship is initially somewhat tentative, but that continued interaction between partners results in cospecialized assets and improves the "information flow between the partners, which may lead to an increase in the performance of the alliance over time" (p. 470). This story predicts that alliance performance should improve over time. The second story posits that the relationship is initially very positive, "since an alliance starts with an initial stock of assets, which may include favorable prior beliefs, goodwill, trust, financial investments, or psychological commitment" (p. 471). This story predicts that "alliance performance should deteriorate over time with the erosion of the initial stock of assets that created the honeymoon period" (p. 472). Because their analysis is cross-sectional, there is also a survivorship effect that implies that older alliances (that are still in existence) will tend to be the more successful ones. Our model is actually consistent with either prediction, because the *ex ante* expected payoffs under private information can be higher or lower than the full-information payoffs, depending on the parametric regime; of course, the more interesting predicted pattern involves declining payoffs over time as private information becomes common knowledge. Deeds and Rothaermel regress a measure of alliance performance on a linear and a quadratic term in the age of the alliance (and other control variables). They find that the linear term is negative and significant ($p < 0.01$) and the quadratic term is positive and significant ($p < 0.01$), implying that alliance performance initially declines, and then improves (if only due to survivorship in the data), with age. Because this is a cross-sectional analysis, it is not conclusive about the performance of a single alliance over time, but it is nevertheless suggestive, and is consistent with the more interesting predicted pattern of our model.

### 1.1 Related Literature

One strand of related literature deals with motivating agents who are subject to moral hazard. In Morrison and Wilhelm (2004), a firm consists of overlapping generations of associates and partners. While they are young, associates obtain skills through mentoring by partners, and skilled associates are promoted to partnership; while they are old, partners mentor associates and then sell their shares in the firm. A partnership that promotes an unskilled associate is revealed as untrustworthy and can command only lower fees from future clients (and thus, the value of a partner's share is reduced). This provides sufficient incentive to motivate partners to engage in the costly process of mentoring associates. Chou (2007) provides a moral-hazard-based model of a partnership in which the key feature is "identity-mixing"; that is, the contribution of one partner to the success of a project may be misattributed to another partner. This modifies each partner's outside option (in bargaining over the proceeds) and can improve investment within the partnership.

In Bar-Isaac (2007), there are overlapping generations of senior and junior agents. It is common knowledge that no one knows the ability of a junior agent, whereas a senior agent knows his own ability. Bar-Isaac shows that if: (1) there is identity mixing so that the client cannot identify which agent was responsible for a successful project, and (2) the senior agent can specify in advance the price at which the junior agent can buy the firm, then the senior agent has an incentive to work in the second period because failure would hurt the reputation of his junior partner, and only a junior agent with a good reputation will be willing to buy the firm. Thus, a senior agent will pair with a junior agent of unknown quality in order to commit himself to exert effort (as he would otherwise shirk). Note that in this model, it is the *common imperfect information* about the junior agent's quality that is being used to leverage the senior agent's effort.

In a single-agent version of our model the more talented type distorts her effort upward (and consequently works too hard) in order to distinguish herself from her less talented alter ego; this is socially wasteful signaling. Several previous single-agent models generate the result that agents work "too hard."[4] Landers et al. (1996) provide

4. Akerlof (1976) provides a multiple-worker model wherein workers work too hard.

a screening model of a law firm, wherein an associate has private information about her type. In order to screen the associate, the partner offers a menu of contracts. The contract designed for the more talented associate involves higher compensation, but hours are distorted upward from the complete-information level. In this case, the excessive effort is due to the associate's private information. In the career concerns model of Holmström (1982/1999) and Dewatripont et al. (1999), both a worker and a firm are symmetrically uncertain about the worker's talent. Because the worker's future compensation depends on the posterior expectation of his talent, he may exert excessive effort in an attempt to bias upward this posterior expectation. In our model each entrepreneur knows her own talent and the unique (refined) equilibrium involves costly effort, providing a perfect signal of that talent; effort also impacts the entrepreneurs' payoffs directly through increasing the value of the output. Moreover, our model involves two entrepreneurs who signal simultaneously to potential buyers.

There is a small literature involving simultaneous signaling in oligopoly markets. In some cases, firms know their rivals' types and use price and advertising to signal quality (e.g., Hertzendorf and Overgaard, 2001; Fluet and Garella, 2002) or use price to signal cost (e.g., Harrington, 1987; Bagwell and Ramey, 1991; Orzach and Tauman, 1996). In other cases, each firm's type is its private information and price is used to signal quality (e.g., Daughety and Reinganum, 2007, 2008)[5] or cost (e.g., Mailath, 1988, 1989; Martin, 1995; Das Varma, 2003). We are not aware of any other simultaneous signaling models in which complementary goods-producers or workers use effort to signal ability.[6]

This paper also contributes to the literature on the effect of incomplete information on welfare. Many simultaneous-move games have Bayesian Nash equilibria with lower surplus than their complete-information analogs. A good example is an optimal auction wherein incomplete information necessitates the use of a reserve price that can result in the seller inefficiently retaining the object. In the case of sequential games, we note that single-agent models involving screening typically result in some inefficiency, and we have already noted that the

5. Both of these papers show how strategic complementarity and incomplete information can soften price competition between oligopolists, and may lead to higher profits (though lower welfare).

6. In independent work, Gervais and Goldstein (2007) consider a partnership model with strategic complementarities and moral hazard in which one partner has a positive self-perception bias regarding the importance of his contribution to the joint output. This bias essentially serves as a credible commitment for the biased agent to choose higher effort; due to the complementarity, the unbiased agent will also increase effort. As a consequence, positive self-perception bias can improve the expected payoffs of both workers. Although some of our results have a similar flavor, it is important to distinguish our fully rational signaling model from their behavioral model.

perfect Bayesian equilibria of single-agent signaling models typically result in lower *ex ante* expected payoffs due to the distortion that is often required to deter mimicry. Several of the multiagent signaling models listed above involve the signaling agents benefiting from the need to signal, but this comes at the expense of other agents in the model (typically consumers; see, e.g., Harrington, 1987; Mailath, 1989; Orzach and Tauman, 1996; Daughety and Reinganum, 2007, 2008). In the present model, however, incomplete information provides perfect Bayesian equilibria whose payoffs represent a Pareto-improvement (not just an increase in total surplus) relative to the complete-information Nash equilibrium payoffs.

Another related strand of this literature involves sequential models in which a team leader signals information about the state of the world to other team members (followers). In Hermalin (1998), a team produces a product whose value depends on each member's effort and the state of the world; that is, $V = \theta\Sigma_n e_n$, where $e_n$ is agent $n$'s effort level and $\theta$ is the state of the world. Team members' effort levels are not strategic complements; rather, each agent has a dominant strategy for providing effort that is increasing in $\theta$. Contracts cannot be written over effort (as is also true in our model). A single team member who knows the state of the world chooses her effort level first; the leader reveals the state of the world by choosing a higher level of effort than she would under symmetric information. The other team members correctly infer $\theta$ and then follow their $\theta$-contingent dominant strategies. Because the leader works harder under asymmetric information and the followers work exactly as hard in both information structures, aggregate welfare is higher but the leader is *worse off* under private information (for any given compensation structure). Thus, private information does not yield a Pareto-improvement. Based on the welfare measure of total surplus, Hermalin remarks (p. 1191) that "the hidden information problem counteracts the teams problem (free-riding)." Komai et al. (2007) and Komai and Stegeman (2007) provide related models that restrict the leader's available strategies so that his action cannot reveal completely the state of the world and show that, for some parameters, the equilibrium under incomplete information represents a Pareto-improvement over complete information. The leader–follower structure and the strategy–space restrictions are critical for this result to hold.

Our model is quite different from these "leadership" models. In the leadership model, the leader serves a coordinating function by influencing the followers' beliefs about the common state of the world. In our model, there is no common state of the world, because each entrepreneur has her own privately known type. Moreover, in our model, the entrepreneurs move simultaneously and thus do not signal to each other; rather, their effort levels provide perfect signals to potential buyers. As in the leadership model, our broad conclusion is that adverse selection can mitigate the moral hazard problem, but the simultaneous choice of effort from unrestricted strategy spaces means that this result does not rely on any coordination arising from sequential moves by the agents.

### 1.2 Plan of the Paper

In Section 2 we develop the basic model and provide the equilibrium under both complete information and incomplete information. Section 3 uses a combination of analytical and numerical techniques[7] to compare the equilibria, both in terms of effort and *ex ante* expected payoffs, under the two information regimes when efforts are strategic complements. We focus on the comparison of truly private information (wherein only each entrepreneur herself knows her talent) with complete information.[8] Section 4 considers two variations on the model developed in Section 3: (a) a sole-entrepreneur version and (b) a two-entrepreneur version wherein the value of the good is the sum of the productivities. Section 5 provides a summary of results and a discussion of possible extensions. There are two appendices: the Appendix contains the derivations and proofs for the most significant aspects of the analysis while the Supporting Appendix[9] contains the more mundane proofs.

## 2. Model Setup and Equilibrium Under Complete and Incomplete Information

We model the problem as a one-shot, three-stage game. In the first stage, Nature chooses a type for each entrepreneur (talent, $t_i$, for entrepreneurs $i = 1, 2$). The type can be high ($t_H$) or low ($t_L$), with

7. The equilibrium payoffs are compared using numerical analysis employing *Mathematica*® 5.2. Copies of the relevant notebooks are available upon request from the authors.

8. One way to think of a signaling model is that it is predicated on the assumption that privately observed information is impossible to disclose credibly. In this case, it is reasonable to assume that neither potential buyers nor the other agent know a given agent's talent. At the other extreme, if privately observed information can be disclosed credibly at no cost, then it *will* be disclosed (because market participants will adopt beliefs that are most detrimental to a nondisclosing agent). In this case, talent will be known by both potential buyers and the other agent. So the comparison of perfectly private versus perfectly public information can be rationalized as the result of polar assumptions regarding the cost of credible information provision.

9. http://www.vanderbilt.edu/Econ/faculty/Daughety/SuppAppforHiddenTalents.pdf

the probability of high being $\lambda$, a positive fraction (i.e., $\Pr\{t_i = t_H\} = \lambda \in (0, 1)$, $i = 1, 2$); Nature's draws are i.i.d. and $0 < t_L < t_H < 1$.[10] Furthermore, let $\bar{t} \equiv \lambda t_H + (1 - \lambda)t_L$; that is, $\bar{t}$ is the expected talent for either entrepreneur. If the game is one of complete information, then in the second stage both entrepreneurs observe the vector of types and each chooses her effort level independently and simultaneously with the other entrepreneur. If the game is one of incomplete information, then each entrepreneur's type is her private information and each chooses her effort level independently and simultaneously with the other entrepreneur.[11] Effort for entrepreneur $i$ is denoted by $e_i$, and may be any nonnegative real number. Each entrepreneur incurs a quadratic cost of exerting effort that depends upon her talent. This cost is given by the expression $(e_i)^2/(4t_i)$, $i = 1, 2$; thus, entrepreneur $i$'s cost of effort is a decreasing function of entrepreneur $i$'s talent. Entrepreneurs act in a noncooperative manner under both complete and incomplete information, as we take both effort and talent to be nonverifiable, so that no contract that induces more cooperative behavior can be written.

Finally, in the third stage, at least two homogeneous potential buyers bid for the completed product. Each buyer has unit demand, with reservation value equal to the actual (in the case of complete information) or inferred (in the case of incomplete information) value of the product. The value of the product, $V$, is the product of the entrepreneurs' productivities, where each entrepreneur's productivity is the sum of her talent and effort. That is, in the case of complete information, entrepreneur $i$'s productivity is given by $P_i = t_i + e_i$ and the value of the product is given by $V = P_iP_j = (t_i + e_i)(t_j + e_j)$. Note that $P_i$ and $P_j$ are imperfect substitutes in production. The limiting case in which they are perfect substitutes (i.e., $V = P_i + P_j$) is considered in Section 4.

In the case of complete information, potential buyers can observe $\{t_i, e_i\}$ for $i = 1, 2$, whereas in the case of incomplete information, potential buyers observe individual effort levels $\{e_i\}$, $i = 1, 2$, and form beliefs about the individual talents based on the observed effort levels, resulting in perceived talents $\{\tilde{t}_i\}$, $i = 1, 2$. In all circumstances the two entrepreneurs are assumed to split the returns from the sale of the

10. One could extend the definitions of the talent values beyond the indicated domain by adding another parameter to the cost function. Because this would add notational complexity without generating additional insights, we will retain the limited domain for the talent levels.

11. One could alternatively consider a model wherein the two entrepreneurs know each others' types, but these are still private information relative to the potential buyer. Both the incentive to shirk and the incentive to signal would still be present in such an analysis. Thus, although we view the current setup as the case of primary interest, we conjecture that similar results would obtain under this modified information structure.

product equally; because demand exceeds supply, the entrepreneurs are able to extract the full perceived value of the product, which is given by $(\tilde{t}_i + e_i)(\tilde{t}_j + e_j)$.

Therefore, the payoff that entrepreneur $i$ receives from the sale of the product, denoted by $u_i(e_i, t_i, \tilde{t}_i \mid e_j, \tilde{t}_j)$, is

$$u_i(e_i, t_i, \tilde{t}_i \mid e_j, \tilde{t}_j) \equiv (\tilde{t}_i + e_i)(\tilde{t}_j + e_j)/2 - (e_i)^2/(4t_i), \quad i, j = 1, 2, j \neq i. \tag{1}$$

That is, entrepreneur $i$'s payoff is a function of her effort, true and perceived types ($t_i$ and $\tilde{t}_i$, respectively), and the other entrepreneur's perceived total productivity (i.e., $\tilde{t}_j + e_j$).

### 2.1 Complete Information

As indicated above, under complete information both talents are observable by each entrepreneur as well as the potential buyers. Thus, potential buyers will bid up the price of the product to its value $V$, so that entrepreneur $i$ chooses $e_i$ to maximize her payoff:

$$u_i(e_i, t_i, t_i \mid e_j, t_j) \equiv (t_i + e_i)(t_j + e_j)/2 - (e_i)^2/(4t_i), \quad i, j = 1, 2, j \neq i. \tag{2}$$

Given that the entrepreneurs are choosing effort levels noncooperatively, a convenient form (abusing notation) for entrepreneur $i$'s payoff is $u_i(e_i, t_i, t_i \mid P_j) = (t_i + e_i)P_j/2 - (e_i)^2/(4t_i)$. Solving entrepreneur $i$'s decision problem:

$$e_i = \arg\max_e u_i(e, t_i, t_i \mid P_j),$$

we obtain $i$'s best-response function, $e_i = t_iP_j = t_i(t_j + e_j)$. Notice that $i$'s best-response function is upward sloping in entrepreneur $j$'s effort level, so that effort levels are strategic complements.

Let $e_{rs}$ be the equilibrium effort level for entrepreneur $i$ when her type is $r$ and the other entrepreneur's type is $s$ ($r, s = H, L$), so that we obtain:

$$e_{rs} = t_r t_s(1 + t_r)/(1 - t_r t_s), \quad r, s = H, L. \tag{3}$$

It is straightforward to show that the equilibrium effort level is increasing in the talent of either entrepreneur. Similarly, it is straightforward to show that the equilibrium payoff for entrepreneur $i$ when her type is $r$ and the other entrepreneur's type is $s$ ($r, s = H, L$), denoted as $u_{rs}$, is

$$u_{rs} = t_r t_s(1 + t_r)(2 + t_s(1 - t_r))/\left[4(1 - t_r t_s)^2\right], \quad r, s = H, L, \tag{4}$$

and that $u_{rs}$ is increasing in the talent of either entrepreneur. With a little algebra one can show that $u_{HL} > u_{LH}$, which along with the previous statement about positive partial derivatives, implies that:

$$u_{HH} > u_{HL} > u_{LH} > u_{LL}. \tag{5}$$

That is, under complete information, an entrepreneur obtains a higher *ex post* equilibrium payoff from being an *H*-type entrepreneur than from being an *L*-type entrepreneur and a higher *ex post* equilibrium payoff when matched with an *H*-type entrepreneur rather than an *L*-type entrepreneur.

Finally, the *ex ante* expected payoff for either entrepreneur under complete information, which we denote as $U^{CI}$, is

$$U^{CI} \equiv \lambda^2 u_{HH} + \lambda(1-\lambda)u_{HL} + (1-\lambda)\lambda u_{LH} + (1-\lambda)^2 u_{LL}. \tag{6}$$

Here, $U^{CI}$ is strictly increasing in $\lambda$ (this is shown in the Supporting Appendix). These *ex ante* expected equilibrium payoffs are the same for each entrepreneur (and, thus, we have dropped the index for the entrepreneur), because Nature's draws for entrepreneur types in stage 1 are i.i.d.

### 2.2 Incomplete Information

As indicated earlier, by incomplete information we mean that each entrepreneur's type is her private information. The equilibrium notion we will employ is perfect Bayesian equilibrium (PBE), and we will focus on symmetric separating equilibria, so this means that we must specify beliefs for the buyers about the entrepreneurs' types for the equilibrium. In equilibrium these beliefs must be correct and allow the types to separate, so that when the entrepreneurs individually and simultaneously choose effort levels in the second stage, each chooses effort so as to maximize her expected payoff, where the expectation is over the other entrepreneur's possible types, and accounts for the beliefs that buyers will hold. Finally, separation will require these choices to respect appropriately defined incentive compatibility constraints.

We have assumed that effort levels are observable to buyers, so that effort levels may signal information about talent to the buyers. Because the entrepreneurs move simultaneously, when talent is private information we assume that each entrepreneur's effort level cannot signal information about the other entrepreneur's type.[12] Because the

12. Fudenberg and Tirole (1991, pp. 332–333) incorporate this restriction on beliefs, which they refer to as "no signaling what you don't know," into their definition of perfect Bayesian equilibrium for a general class of abstract games, of which ours is a special case.

entrepreneurs are identical *ex ante*, we will treat them identically and therefore restrict beliefs to be independent of identity. Thus, the perceived level of talent for entrepreneur $i$, $\tilde{t}_i$, is based on entrepreneur $i$'s observed effort, $e_i$, and is captured by the notation $\mu(e_i)$, where $\mu(\cdot) \in \{t_H, t_L\}$. Suppose that entrepreneur $j$ uses a separating equilibrium strategy denoted as $e^*(t)$; that is, $e^*(t_L) \neq e^*(t_H)$. Then entrepreneur $i$ predicts that $j$'s productivity will be, and the buyers will perceive it to be, $(t_H + e^*(t_H))$ with probability $\lambda$ and $(t_L + e^*(t_L))$ with probability $(1 - \lambda)$. Therefore, entrepreneur $i$ chooses an effort level so as to maximize her expected payoff, where the expectation is taken over entrepreneur $j$'s type (this will be indicated via the operator $E_j$ in what follows).

Let $P^* \equiv E_j\ [t_j + e^*(t_j)]$; then entrepreneur $i$'s payoff function, incorporating buyer beliefs and the separating equilibrium strategy adopted by entrepreneur $j$, can be re-expressed as

$$u_i(e_i, t_i, \mu(e_i) \mid P^*) \equiv (\mu(e_i) + e_i)E_j[t_j + e^*(t_j)]/2 - (e_i)^2/(4t_i)$$
$$= (\mu(e_i) + e_i)P^*/2 - (e_i)^2/(4t_i).$$

In order for $e^*(t)$ to be part of a separating equilibrium (i.e., in order for entrepreneur $i$ to use $e^*(t_i)$ as a best response), it must be true that types are as well-off choosing revealing levels of effort as they could be by choosing some other level of effort that may induce consumers to believe that entrepreneur $i$ is of the alternative type. Formally, we write this as requiring equilibrium effort levels $(e_H, e_L)$ such that the following incentive compatibility and consistency-of-beliefs conditions hold:

$$u_i(e_H, t_H, t_H \mid P^*) \geq \max_e u_i(e, t_H, \mu(e) \mid P^*) \quad i = 1, 2; \tag{7a}$$

$$u_i(e_L, t_L, t_L \mid P^*) \geq \max_e u_i(e, t_L, \mu(e) \mid P^*) \quad i, = 1, 2; \tag{7b}$$

$$t_H = \mu(e_H) = \mu(e^*(t_H)) \text{ and } t_L = \mu(e_L) = \mu(e^*(t_L)). \tag{7c}$$

Thus, in the symmetric separating PBE, type $H$ (respectively, type $L$) can reveal her type via effort level $e_H$ (respectively, $e_L$). Note further that conditions (7a) and (7b) also serve to define best-response functions, when viewed as what entrepreneur $i$ should choose *given* the expected productivity of entrepreneur $j$ ($P^*$). Therefore, because this is a symmetric equilibrium, one further requirement is that this conjectured expected productivity must be correct in equilibrium:

$$P^* = \lambda(t_H + e_H) + (1 - \lambda)(t_L + e_L). \tag{7d}$$

That is, the expected productivity for each entrepreneur in the conditioning statement for the incentive compatibility constraints (7a) and (7b) must be the expected value of the productivities that can arise in equilibrium. We now use the discussion above to define formally the equilibrium of interest.

**DEFINITION***: A symmetric separating perfect Bayesian equilibrium consists of a pair of effort levels* $(e_H^*, e_L^*)$ *and beliefs* $\mu^*(e)$ *such that conditions (7a)–(7d) are satisfied.*

In what follows we will show that there is always a unique (refined) symmetric separating PBE. The nature and form of the PBE depend upon the parameters of the model. In one type of PBE, the *H*-type entrepreneur will engage in the sort of distortion familiar to users of signaling models: effort will be distorted upward so that the *L* type will not find it profitable to mimic the *H* type. For example, this will occur if the two levels of talent are (in a sense to be described below) sufficiently close together. In the second type of PBE, there will be no need for the *H* type to distort her effort level in order to separate from the *L* type. We refer to the first type of PBE as a "distortionary" PBE (DPBE) and the second type of PBE as a "non-distortionary" (NPBE).[13] It is straightforward to show that the NPBE is identical to the Bayesian Nash equilibrium (BNE) of a game similar to the one we have described above, except that at the end of stage 2 the types are revealed exogenously, so that potential buyers know the type of each entrepreneur before buying.

We find (computationally) that these BNE always (i.e., throughout the parameter space) involve welfare reductions in the sense that both entrepreneurs are worse off than they would be in the complete-information Nash equilibrium. Thus, one effect of private information is to prevent each entrepreneur from tailoring her strategy to her partner's type, which reduces the *ex ante* expected equilibrium payoffs. A countervailing effect, which is only present in the DPBE, is that the entrepreneurs distort their efforts upward. We find that a significant subset of the DPBE reflect a Pareto-improvement relative to the complete-information Nash equilibrium.

The proof of the following proposition is given in the Appendix; the uniqueness of the equilibrium is the result of the application of the intuitive criterion (Cho and Kreps, 1987). The superscript *D* refers to the DPBE, while the superscript *N* refers to the NPBE.

13. Both types of equilibrium (NPBE and DPBE) are, in fact, *distorted* relative to the complete-information equilibrium; the distortion to which we refer via the letters *N* and *D* is relative to the complete-information best response to the other entrepreneur's expected productivity (see the Appendix for details).

**Proposition 1***: A unique (refined) symmetric separating PBE always exists; it is defined implicitly by the following (parameter-dependent) equations.*

$$P^* = \begin{cases} P^D = \bar{t} + t_L P^D + \lambda\left[2t_L(t_H - t_L)P^D\right]^{1/2} & \textit{when } \bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L) \\ P^N \equiv \bar{t}/(1-\bar{t}) & \textit{when } \bar{t}/(1-\bar{t}) \geq 2t_L/(t_H - t_L) \end{cases} \tag{8a}$$

$$e_H^* = \begin{cases} e_H^D \equiv t_L P^D + \left[2t_L(t_H - t_L)P^D\right]^{1/2} & \textit{when } \bar{t}/(1-\bar{t}) < 2t_L(t_H - t_L) \\ e_H^N \equiv t_H P^N & \textit{when } \bar{t}/(1-\bar{t}) \geq 2t_L(t_H - t_L) \end{cases} \tag{8b}$$

$$e_L^* = t_L P^* \tag{8c}$$

$$\mu^*(e) = t_H \quad \textit{if } e \geq e_H^*, \quad \textit{else } \mu^*(e) = t_L. \tag{8d}$$

The explicit formula for $P^D$ is provided in the Appendix; recall that $\bar{t}$ is the (prior) mean talent.

Proposition 1 formally presents the two points made earlier. First, there always exists a unique (refined) symmetric separating PBE. Second, its nature changes as a function of whether the $H$-type entrepreneur must engage in distortion to actively deter mimicry by the $L$-type entrepreneur. When $\bar{t}/(1-\bar{t}) \geq 2t_L/(t_H - t_L)$, the parameters of the game ($\lambda$, $t_H$, and $t_L$) allow the $H$ type to follow her undistorted best-response function without fear of mimicry (the $L$ type can always follow her undistorted best-response function without fear of mimicry). These undistorted best-response functions are given by $e_r^* = t_r P^N$, $r = H, L$, where the superscript "$N$" denotes the fact that no distortion (relative to the best-response function) is needed to signal type. Notice that the *form* of the best-response functions in this case is similar to the form of the best-response functions in the complete-information case (where $e_i = t_i P_j$), though the actual value of $P^N$ is different from $P_j$. That is, the entrepreneur employs the complete-information best response to the expected productivity of the other entrepreneur. Substituting the equilibrium value of $P^N$ immediately yields the equilibrium effort levels $e_r^* = t_r \bar{t}/(1-\bar{t})$, $r = H, L$. It is straightforward to show that, in any NPBE, $P^N$, $e_H^N$, and $e_L^N$ ($\equiv e_L^* = t_L P^N$) are all increasing functions of $\lambda$, $t_H$, and $t_L$. Thus (for example), an exogenous increase in individual talents increases noncooperative equilibrium effort levels.

When the $L$-type entrepreneur would mimic the $H$-type entrepreneur's undistorted best response (in order to be taken as an $H$ type), then the $H$-type entrepreneur needs to distort her effort upward (relative to the complete-information best response to the expected productivity of the other entrepreneur) in order to separate from the signal sent by the $L$-type entrepreneur: that is, the $H$ type distorts so as to send a signal that makes the $L$ type indifferent between mimicking and choosing an effort based on her own (NPBE) best response. These equilibria occur when $\bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L)$. As can be seen from this inequality, one way this can occur is when $t_H$ is not very different from $t_L$. Alternatively, for any $(t_H, t_L)$ combination, there is a sufficiently small $\lambda$ such that this condition will be met. Much of this will be illustrated below in Figure 1.

As with the NPBE, the comparative statics of the DPBE expressions for $P^D$, $e_H^D$, and $e_L^D$ ($\equiv e_L^* = t_L P^D$) with respect to $\lambda$ and $t_H$ are straightforward (all three equilibrium values are increasing in these two parameters); unfortunately, this is not true for the effect of changes in $t_L$, as the algebra precludes providing a clear sign for these derivatives. We formalize the foregoing comparative statics results in Proposition 2 below.

**PROPOSITION 2:**

(i) *In any DPBE, $e_r^*$, $r = H, L$, is increasing in $\lambda$ and $t_H$.*
(ii) *In any NPBE, $e_r^*$, $r = H, L$, is increasing in $\lambda$, $t_H$, and $t_L$.*

Typically, in monopoly signaling models, the only aspect of the prior distribution that affects the separating equilibrium is the distribution's support.[14] Here, however, an increase in the likelihood that an entrepreneur has the high level of talent ($\lambda$) increases effort by both possible types of entrepreneur. This is because of the fact that effort levels are strategic complements, and the best response depends upon the expected value of the other entrepreneur's type. Therefore, the higher the expected value of entrepreneur $j$'s type (as a result of increasing $\lambda$), the higher is entrepreneur $i$'s best response. Thus, the separating equilibrium is responsive not only to the support of the prior distribution over the type space, but also to the details (here, the magnitude of $\lambda$) of the prior as well.[15] This technical dependence of the incomplete-information equilibria on $\lambda$ provides a further avenue of exploration that we consider later in the paper. An economic implication is that regulatory procedures, such as licensing, educational requirements, or certification, which result in an exogenous increase in the likelihood of high-talent entrepreneurs being in the pool (i.e., increased $\lambda$), increases expected product quality and the entrepreneurs'

14. A few exceptions do exist; see Matthews and Mirman (1983), Caillaud and Hermalin (1993), and Daughety and Reinganum (1995, 2005).

15. Other models with this information structure and multiple players also obtain this kind of prior dependence; see Mailath (1988, 1989), Das Varma (2003), and Daughety and Reinganum (2007, 2008).

expected efforts; as will be seen below, this also leads to an increase in their expected payoffs.

In the second stage of the game each entrepreneur knows her type, but not the other entrepreneur's type, leading to the *interim* equilibrium payoffs for the high and low types, $U_H^{\mathrm{II}}$ and $U_L^{\mathrm{II}}$, respectively:

$$U_H^{\mathrm{II}} \equiv \lambda u_i\left(e_H^*, t_H, t_H \mid e_H^*, t_H\right) + (1-\lambda) u_i\left(e_H^*, t_H, t_H \mid e_L^*, t_L\right); \tag{9a}$$

and

$$U_L^{\mathrm{II}} \equiv \lambda u_i\left(e_L^*, t_L, t_L \mid e_H^*, t_H\right) + (1-\lambda) u_i\left(e_L^*, t_L, t_L \mid e_L^*, t_L\right). \tag{9b}$$

Thus, the *ex ante* expected equilibrium payoff to an entrepreneur under incomplete information, denoted by $U^{\mathrm{II}}$, is

$$U^{\mathrm{II}} \equiv \lambda U_H^{\mathrm{II}} + (1-\lambda) U_L^{\mathrm{II}}. \tag{10}$$

Its exact form is provided in the Appendix, along with the proof of the following proposition providing comparative statics results.

**PROPOSITION 3***: In all symmetric separating perfect Bayesian equilibria:*

(i) $U_H^{\mathrm{II}} > U_L^{\mathrm{II}}$;
(ii) $U_H^{\mathrm{II}}$ *and* $U_L^{\mathrm{II}}$ *are increasing functions of* $\lambda$;
(iii) $U^{II}$ *is an increasing function of* $\lambda$.

Thus, from an *interim* equilibrium perspective, it is always better to be an $H$ type than to be an $L$ type, and the *interim* equilibrium payoffs for both types are increasing in the prior probability that the other entrepreneur will be an $H$ type. This is also a result of the interplay between incomplete information and the strategic complementarity of effort levels. Finally, given the foregoing, the third result follows, namely that the *ex ante* expected payoff from participating in the incomplete-information game is increasing in the prior probability that any entrepreneur is an $H$ type.

We now explore the geometry of the parameter space (i.e., the $\lambda$, $t_H$, and $t_L$ values) that produces the two variants of equilibria; we will illustrate this in $(t_{\mathrm{H}}, t_{\mathrm{L}})$-space. Using the condition $\bar{t}/(1-\bar{t}) = 2t_{\mathrm{L}}/(t_H - t_L)$, which defines the boundary between DPBE and NPBE, and solving for $\lambda$ as a function of $t_H$ and $t_L$, we define $\Lambda(t_H, t_L)$ as

$$\Lambda(t_{\mathrm{H}}, t_{\mathrm{L}}) \equiv t_{\mathrm{L}}(2 - (t_{\mathrm{H}} + t_{\mathrm{L}}))/((t_{\mathrm{H}} + t_{\mathrm{L}})(t_{\mathrm{H}} - t_{\mathrm{L}})). \tag{11}$$

It is straightforward to show that $\Lambda(t_H, t_L)$ is increasing in $t_L$ (see the Supporting Appendix). Recall that $t_L < t_H$ and let:

$$D(t_H, t_L) \equiv \{\lambda \in (0,1) \mid \lambda < \min[\Lambda(t_H, t_L), 1]\}; \tag{12a}$$

and

$$N(t_H, t_L) \equiv \{\lambda \in (0, 1) \mid \lambda \geq \min[\Lambda(t_H, t_L), 1]\}. \tag{12b}$$

The sets $D(t_H, t_L)$ and $N(t_H, t_L)$ partition the possible values of $\lambda$ into those that will, at the given point $(t_H, t_L)$, result in, respectively, a DPBE or an NPBE; note that these sets are disjoint, and $D(t_H, t_L)$ is always nonempty whereas $N(t_H, t_L)$ could be empty. This leads to the following proposition that formalizes the foregoing.

**PROPOSITION 4:**

(i) *For the portion of the* $(t_H, t_L)$ *parameter space wherein* $\Lambda(t_H, t_L) \geq 1$*, every possible prior* $\lambda \in (0, 1)$ *leads to a DPBE.*
(ii) *For values of* $t_H$ *and* $t_L$ *such that* $0 < \Lambda(t_H, t_L) < 1$*, a DPBE prevails when* $\lambda < \Lambda(t_H, t_L)$*, whereas an NPBE prevails when* $\lambda \geq \Lambda(t_H, t_L)$*.*

Figure 1 below illustrates the regions wherein DBPE occur in the $(t_H, t_L)$ space, where $\bar{\lambda} \equiv \min[\Lambda(t_H, t_L), 1]$ denotes the upper bound on the values of the prior $\lambda$ that yield a DPBE. In the innermost "lens" (the region bounded by the 45-degree line $t_L = t_H$ and the curve labeled $\bar{\lambda} = 1$), all combinations of $t_L$ and $t_H$ values result in DPBE for all possible prior probabilities ($\lambda$) that an entrepreneur's talent is $t_H$, as Proposition 4 indicates. Along the curve labeled $\bar{\lambda} = 0.5$ the equilibrium is a DPBE if the prior probability that a partner is an $H$ type is less than 0.5, but it is an NPBE should $\lambda$ be at least 0.5. Finally, a yet lower curve labeled $\bar{\lambda} = 0.25$ is displayed, which contains combinations of $t_L$ and $t_H$ values that yield a DPBE if $\lambda < 0.25$ and an NPBE if $\lambda \geq 0.25$. The figure reflects the previously noted monotonicity of $\Lambda(t_H, t_L)$ in $t_L$: for a fixed value of $t_H$, increasing $t_L$ is associated with an increase in the set of $\lambda$ values for which a DPBE prevails (i.e., $D(t_H, t_L)$ "grows" as $t_L$ increases), until we reach the curve labeled $\bar{\lambda} = 1$, beyond which only DPBE exist. Alternatively put, $N(t_H, t_L)$ gets "crowded out" as $t_L$ increases (given $t_H$); this property will be employed in the numerical analysis in the next section.

Intuitively, what Figure 1 illustrates is that when the high- and low-talent values are somewhat close, it is not very costly for the $L$-type entrepreneur to mimic the $H$-type entrepreneur's higher effort. In this case, the $H$-type entrepreneur will need to distort effort upward in order to separate from the $L$-type entrepreneur; thus the equilibrium is a DPBE. However, when $t_L$ and $t_H$ are sufficiently far apart, and the prior probability that the other entrepreneur is an $H$ type is sufficiently high, then the (refined) equilibrium is an NPBE.

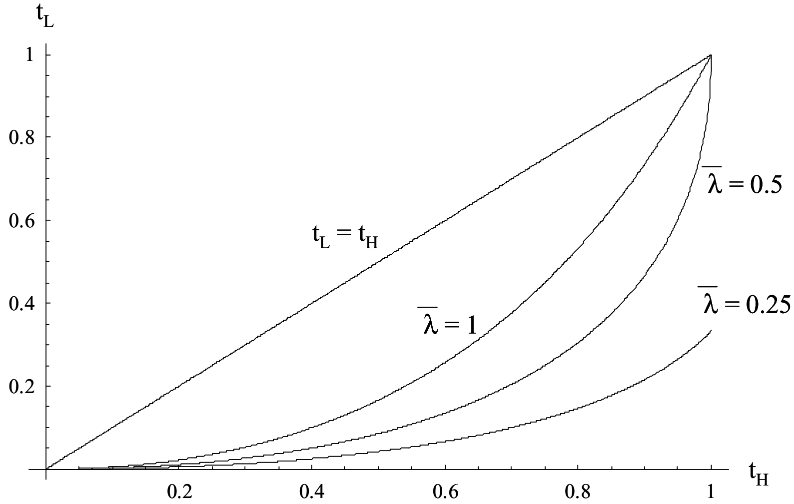


*FIGURE 1. REGIONS GUARANTEEING DPBES WHEN* $\lambda \in (0, \bar{\lambda})$

## 3. Comparing Equilibrium Results Under Complete Information and Incomplete Information

In this section we use both analytical and numerical techniques to examine the relationship between the complete-information and incomplete-information entrepreneurship equilibria.

### 3.1 Equilibrium Effort Levels

Recall that the complete-information *ex post* equilibrium effort levels, as specified in result (3) above, were denoted as $e_{rs}$, $r, s = H, L$, and that the incomplete-information equilibrium effort levels were denoted as $e_r^N$, $r = H, L$ in the NPBE case and as $e_r^D$, $r = H, L$ in the DPBE case. The following proposition (proved in the Appendix) provides comparisons of these equilibrium effort levels between the two information conditions.

**Proposition 5***: Comparison of equilibrium effort levels, with* $0 < t_L < t_H < 1$.

*(i) At any* $(t_H, t_L)$: $e_{HH} > e_{HL} > e_{LH} > e_{LL}$.

*(ii) At any* $(t_H, t_L)$ *such that* $N(t_H, t_L)$ *is nonempty, and for all* $\lambda \in \mathrm{N}(t_H, t_L)$:

*(a)* $e_{HH} > e_H^N > e_{HL}$;

*(b)* $e_L^N > e_{LL}$;

*(c)* $e_L^N (>=<) e_{LH}$ *as* $\lambda\ (> = <) 1/(1 + t_H)$.

*(iii) At any* $(t_H, t_L)$: $e_L^D > e_{LL}$ *for all* $\lambda \in D\ (t_H, t_L)$.

*(iv) At any* $(t_H, t_L)$ *such that* $\Lambda(t_H, t_L) > 1$: $\lim_{\lambda\to 1} e_H^D > e_{HH}$ *and* $\lim_{\lambda\to 1} e_L^D > e_{LH}$.

Proposition 5 provides a number of results. As indicated in part (i), the complete-information equilibrium effort levels are always ordered as intuition would suggest: not only does an improvement in an entrepreneur's type increase the equilibrium effort for either entrepreneur (as pointed out below equation (3) earlier), effort is also increasing as one might expect in regards to the cross-terms (i.e., $e_{HL} > e_{LH}$). Part (ii) concerns comparisons between the complete-information equilibrium effort levels and those in an NPBE. These results are followed by the comparisons between complete-information effort levels and those in a DPBE in parts (iii) and (iv).

Consider the results concerning NPBE. Although there are portions of the parameter space wherein the $L$-type's effort is higher under incomplete information than the relevant comparison effort level under complete information,[16] notice that the $H$-type's effort level in an NPBE is always less than $e_{HH}$. This contrasts with the $H$ type results in the innermost lens of Figure 1 (wherein all equilibria are DPBE). In that portion of the parameter space, part (iv) of Proposition 5 indicates that if the prior probability of an $H$ type is high enough, then incomplete information results in a higher effort level for type $H$ than would obtain under complete information ($\lim_{\lambda\to 1} e_H^D > e_{HH}$, and due to part (i), $\lim_{\lambda\to 1} e_H^D > e_{HL}$). This upward distortion of effort under incomplete information is also true in the same portion of the parameter space for the $L$ type. Thus, when $\lambda$ is high enough, there are always DPBE (i.e., when $\Lambda(t_H, t_L) > 1$) wherein the effort levels for both types will be higher under incomplete information than under complete information.

## 3.2 Equilibrium Payoff Levels

Recall the definitions of the *ex ante* expected equilibrium payoff under incomplete information (see (10) above), $U^{\text{II}}$, and the *ex ante* expected equilibrium payoff under complete information (see (6) above), $U^{CI}$. It is straightforward to show that $\lim_{\lambda\to 0} U^{\text{II}} = U^{CI}$. This, and the effort results above, suggest that private information may increase *ex ante* expected equilibrium payoffs for the entrepreneurs, at least in some portion of the parameter space (of course, in this model, the buyer is always fully extracted). For example, when $(t_H, t_L) = (0.75, 0.60)$, then $\Lambda(t_H, t_L) = 1.92593$; thus, the point $(t_H, t_L) = (0.75, 0.60)$ is in the

16. It may seem intuitive that the *interim* effort levels ($e_H^N$ and $e_L^N$) should always fall between the corresponding complete-information effort levels for a given type, but this is not the case (see the proof of this proposition in the Appendix). In particular, $e_H^N > e_{HL}$ only if $\lambda$ is sufficiently large, whereas $e_L^N < e_{LH}$ only if $\lambda$ is sufficiently small.

innermost lens in Figure 1. A graph of the difference $U^{\mathrm{II}} - U^{CI}$ is shown in Figure 2. Here, $U^{\mathrm{II}} > U^{CI}$ for all values of $\lambda > 0$. Therefore, at this particular point (and, clearly, in a neighborhood of this point), private information is Pareto-improving, for *all* possible priors, in comparison with complete information.

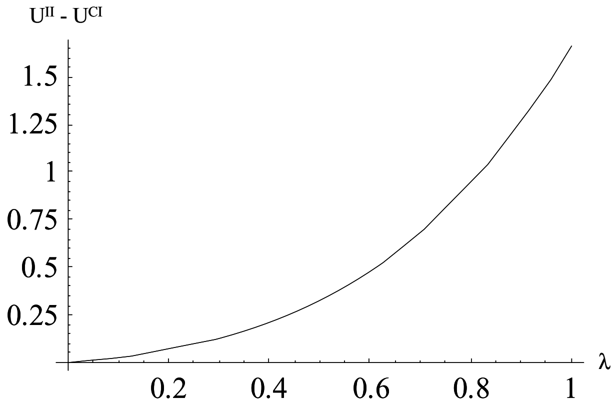


*FIGURE 2. THE GAIN FROM PRIVATE INFORMATION AT* $(t_H, t_L) = (0.75, 0.60)$

A second computation shows that $U^{\mathrm{II}} > U^{CI}$ does not hold everywhere in the innermost lens (for all $\lambda$). For example, when $(t_H, t_L) = (0.75, 0.50)$, then $\Lambda(t_H, t_L) = 1.2$; thus, this point is also in the innermost lens in Figure 1. However, as Figure 3 shows, $U^{\mathrm{II}} > U^{CI}$ only for sufficiently high values of $\lambda$ (here, approximately for $\lambda > 0.63$). Therefore, at some points in the innermost lens, although

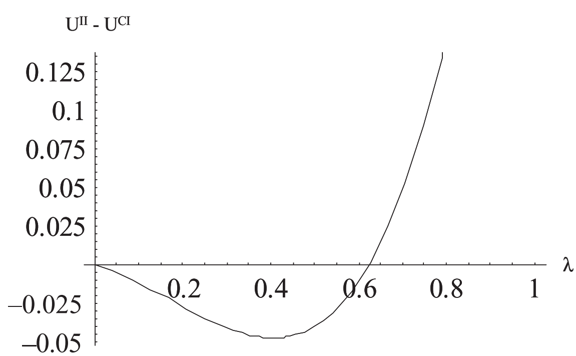


*FIGURE 3. THE GAIN/LOSS FROM PRIVATE INFORMATION AT* $(t_H, t_L) = (0.75, 0.50)$

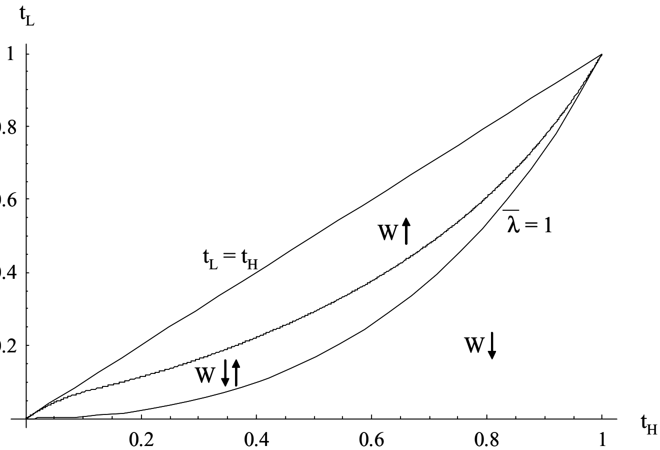


*FIGURE 4. NUMERICAL ANALYSIS OF THE EFFECT OF PRIVATE INFORMATION ON WELFARE*

not all priors lead to the Pareto-superiority of incomplete information, an open dense set of priors will do so (i.e., for high enough values of $\lambda$).

Unfortunately, we have found it impossible to provide a complete (algebraic) characterization of the region wherein $U^{\text{II}} > U^{CI}$ for all $\lambda$. In what follows we use computational techniques to address two questions. First, what is the subregion wherein $U^{\text{II}} > U^{CI}$ for all values of $\lambda$, and second, what happens in the rest of the parameter space? We address the second issue first: a numerical screen of the parameter space outside of the innermost lens in Figure 1 (i.e., the region wherein N ($t_{\text{H}}$, $t_{\text{L}}$) is nonempty at each point), indicates that $U^{\text{II}} < U^{CI}$: incomplete information is welfare diminishing for all $\lambda > 0$. These results, and a similar exploration of the lens generated by the curve $\bar{\lambda} = 1$, yields Figure 4.

In Figure 4 the region labeled "W↑" contains those ($t_H$, $t_L$) pairs wherein the incomplete-information (DBPE) payoffs to the entrepreneurs always exceed their complete-information payoffs: $U^{\text{II}} > U^{CI}$ for all possible $\lambda \in (0, 1)$. The region labeled "W↓" (all of which lies outside of the innermost lens defined by curve $\bar{\lambda} = 1$) contains those ($t_{\text{H}}$, $t_{\text{L}}$) pairs wherein the incomplete-information (DBPE and NPBE) payoffs are always less than those of the complete-information equilibrium: $U^{\text{II}} < U^{CI}$ for all possible $\lambda > 0$. Finally, the subregion inside the innermost lens, which is labeled "W↓↑," contains those ($t_{\text{H}}$, $t_{\text{L}}$) pairs wherein the result of comparing the incomplete-information (DBPE) payoffs and the complete-information payoffs depends upon the prior in question: a sufficiently high prior on the *H* type results in

$U^{\text{II}} > U^{\text{CI}}$. In other words, points in this region generate graphs similar to Figure 3 above.

Thus, the numerical analysis suggests the following conjecture comparing the *ex ante* expected payoffs (recall that, by direct computation, we know that there are $(t_{\text{H}}, t_{\text{L}})$ pairs such that $U^{\text{II}} > U^{CI}$ for all possible values of $\lambda$).

CONJECTURE (based on numerical analysis):

(i) A necessary, but not sufficient, condition for $U^{\text{II}} > U^{CI}$ *for all* possible prior probabilities of an $H$ type is that $\Lambda(t_{\text{H}}, t_{\text{L}}) > 1$;
(ii) A necessary and sufficient condition for $U^{\text{II}} > U^{CI}$ *for some* $\lambda > 0$ is that $\Lambda(t_H, t_L) > 1$.

## 4. Relaxing Strategic Complementarity

In this section we consider two variations on the previous analysis. First, we consider a model in which the product is produced by a sole entrepreneur. Then we return to the two-entrepreneur setting above, but now we assume that the value of the output is the sum of the productivities rather than their product. In each of these cases, entrepreneurs follow dominant strategies, so that there is no strategic complementarity present.

### 4.1 The Sole-Entrepreneur Model

The sole-entrepreneur's payoff is

$$u(e, t, \tilde{t}) \equiv (\tilde{t} + e) - (e)^2/(4t),$$

where the true type $t$ and the perceived type $\tilde{t}$ need not be the same. The complete-information optimal effort levels are given by $e_r = 2t_r$, $r = H, L$. The details of the analysis under incomplete information are provided in the Supporting Appendix where we show that the $H$-type's separating equilibrium effort level is given by $e^*_H = 2t_H$ if $t_L \leq t_H/2$ and $e^*_H = 2t_L + 2[t_L(t_H - t_L)]^{1/2}$ if $t_L > t_H/2$. Note that $2t_L + 2[t_L(t_H - t_L)]^{1/2} > 2t_H$, so that when $t_L > \text{t}_H/2$, the equilibrium requires the $H$ type to distort effort to a level in excess of the complete-information level, which means that the $H$ type chooses an effort level where profits for such a type are falling in effort. The $L$-type's separating equilibrium effort level is given by $e^*_L = 2t_L$. Further, note that the separating equilibrium effort levels depend on the support of the prior distribution, but not on $\lambda$.

The point is that for a sole entrepreneur, *ex ante* expected equilibrium payoffs are never greater under incomplete information than under complete information and, for portions of the parameter space wherein a signaling distortion is part of the equilibrium, signaling is wasteful in the sense that the sole entrepreneur's *ex ante* expected equilibrium payoff is lower in the incomplete-information case than would be achieved in the complete-information case. This is a familiar result in single-agent signaling models.

### 4.2 Two-Entrepreneur Model When Productivities Are Perfect Substitutes

It is straightforward to extend the sole-entrepreneur model to the case of two entrepreneurs whose productivities are perfect substitutes in production; that is, the value of the product is $V = P_i + P_j = t_i + e_i + t_j + e_j$. In this case, their effort choices are strategically independent. The payoff function for entrepreneur $i$ is

$$u_i(e_i, t_i, \tilde{t}_i \mid e_j, \tilde{t}_j) \equiv (\tilde{t}_i + e_i + \tilde{t}_j + e_j)/2 - (e_i)^2/(4t_i), \quad i, j = 1, 2, j \neq i,$$

where $\tilde{t}_i$ and $\tilde{t}_j$ denote entrepreneur $i$'s and entrepreneur $j$'s perceived types. Under complete information, an entrepreneur of type $r$ has a dominant strategy to choose $e_r = t_r$, $r = H, L$. The details of the analysis under incomplete information are given in the Supporting Appendix, where we show that the $L$-type entrepreneur will choose her complete-information optimal effort ($e_L^* = t_L$),whereas the $H$-type's separating equilibrium effort level is given by $e_H^* = t_H$ if $t_L \leq t_H/3$ and $e_H^* = t_L + [2t_L(t_H - t_L)]^{1/2}$ if $t_L > t_H/3$.

Thus, if $t_L \leq t_H/3$ then there is no difference between the *ex ante* expected equilibrium payoffs under complete versus incomplete information (because the $H$ type need not distort her effort to signal her type). On the other hand, if $t_L > t_H/3$ then the $H$ type distorts her effort upward, but to a level that does not exceed the joint profit-maximizing complete-information effort level, that is, $e_H^* \in (t_H, 2t_H)$. It is tedious but straightforward to show that when distortion is needed to signal type $H$, then each entrepreneur's *ex ante* expected equilibrium payoff under incomplete information is higher than the *ex ante* expected equilibrium payoff when strategies are chosen under complete information.

## 5. Summary of Results and Directions for Extension

The foregoing discussion can be summarized as follows. Our model is one of multiagent simultaneous signaling; each entrepreneur is privately informed about her own talent, but she is not informed about the other entrepreneur's talent (moreover, buyers do not know either entrepreneur's talent). Entrepreneurs then simultaneously choose effort levels, followed by the sale of the product. We find a number of results. First, there are portions of the ($t_H$, $t_L$, $\lambda$)-space wherein the *ex ante* expected equilibrium payoff to each entrepreneur is higher, and buyers are no worse off, when there is signaling than when there is complete information. This is because the value of the entrepreneurs' product is increased in this region (relative to the complete-information product), so private information is Pareto-improving in this region. Second, this result occurs only in the "distortionary" equilibria, and not in any of the NPBE (which coincide with the BNE of the modified game wherein potential buyers are exogenously informed of the entrepreneurs' types before purchase). This suggests a critical role for sequential formation of buyer beliefs based on entrepreneurs' actions, and for distortionary signaling by entrepreneurs to influence those beliefs, in creating conditions wherein private information is Pareto-improving. Third, under the multiplicative payoff specification, this result holds for *all* possible priors in an open, dense set of points within the innermost lens (the W↑ set in Figure 4). Moreover, it appears (again, via the numerical analysis), that the residual set (the W↓↑ set in Figure 4) yields Pareto-improving equilibria for a subset of possible prior distributions. Under the additive payoff specification, private information is Pareto-improving for all ($t_H$, $t_L$, $\lambda$) combinations wherein $t_L > t_H/3$. Thus, the combination of simultaneous-move strategic behavior and incomplete information can lead to conditions wherein the "problem" of adverse selection actually *mitigates* the problem of moral hazard.

Taken together, these results suggest circumstances under which one should expect exceptional performance from an entrepreneur-based market that, in its early stages, arguably exhibits private information. Moreover, under these circumstances, as information about the entrepreneurs' talent in this particular venture becomes common knowledge, the market's performance and the entrepreneurs' payoffs may deteriorate; this appears to be qualitatively consistent with the empirical findings of Deeds and Rothaermel (2003). As a consequence, a team of entrepreneurs might find it desirable to abandon the mature market they have created in search of another entrepreneurial venture (wherein their types are effectively re-drawn).

There are several directions of possible extension. One direction involves exploring the effects of changing the number of entrepreneurs. In this paper we show that expanding the number of entrepreneurs from one to two creates a region of Pareto-improving private information (this was true for both the multiplicative and the additive payoff specifications). What happens as one further expands the number of entrepreneurs? A second direction would examine what happens if entrepreneurs share unequally the product's value. For example, what are the incentive effects of shifting the design to that of a winner-take-all tournament?

Finally, a third direction involves using a variation of this basic model to analyze search and matching with two heterogeneous populations of entrepreneurs with complementary skills. The nature of the heterogeneity would be in terms of $\lambda$, the likelihood that the entrepreneur has high talent. Upon meeting, entrepreneurs could observe each other's respective values of $\lambda$ (but not the other entrepreneur's actual talent), and then decide either to pair up and produce, or to search again. In Proposition 3, we found that, from an *interim* equilibrium perspective, it is always better to be an $H$ type than to be an $L$ type, and the *interim* equilibrium payoffs for both types are increasing in the prior probability that the other entrepreneur will be an $H$ type. From this we conjecture (though the basic model would have to be re-cast and re-solved with entrepreneur-specific values of $\lambda$) that the matching model would involve positive assortative matching. Another return to solving the model with entrepreneur-specific values of $\lambda$ is that this would readily allow for an earlier stage wherein each entrepreneur may be able to make an investment that influences the prior distribution over her talent level, prior to engaging in search and matching or production.

## Appendix

*Proof of Proposition 1.*

### Overview

This proposition is proved by verifying that the strategies described in the proposition satisfy (7a)–(7d) and that they uniquely survive refinement using the intuitive criterion. The proof is by construction, because the strategies stated in the proposition are derived by (i) requiring that constraints (7a)–(7b) be satisfied; (ii) applying the refinement argument to the set of candidate best responses to select a unique candidate pair; (iii) using constraint (7d) to solve for $P^*$, thus generating the equilibrium strategies $e_H^*$ and $e_L^*$; and (iv) associating the proper beliefs (7c) with the equilibrium strategies. Thus, the strategies described in the proposition satisfy (7a)–(7d) and they uniquely survive refinement using the intuitive criterion.

### Solution of Incentive Compatibility Constraints

Recall the incentive constraints (7a) and (7b). If neither type had to worry about mimicry by the other type, then an entrepreneur's best response would be given by $\rho_L^N(P^*) = t_L P^* = \text{argmax}_e\, u_i(e, t_L, \cdot \mid P^*)$ for an $L$ type and by $\rho_H^N(P^*) = t_H P^* = \text{argmax}_e\, u_i(e, t_H, \cdot \mid P^*)$ for an $H$ type. We refer to these as the "non-distortionary best-response" functions for $L$ and $H$, respectively. Note two things. First, "non-distortionary" here means simply that the entrepreneur's best response reflects what she would choose absent any risk of mimicry; equilibrium effort by both partners in *any* PBE will be distorted relative to their complete-information counterparts. Second, the nondistortionary best-response functions depend only on the entrepreneur's own true type and the other entrepreneur's productivity, and *not* on the entrepreneur's own perceived type (though the payoffs do depend on own perceived type).

Because entrepreneur $i$'s payoff is increasing in both $\tilde{t}_i$ and $t_i$, it is always preferable to be perceived as an $H$ type given any true type, and it is always preferable to be an $H$ type given any perceived type. Thus, no entrepreneur would be willing to deviate from her nondistortionary best response in order to be perceived as an $L$ type. On the other hand, either entrepreneur would be willing to deviate from her nondistortionary best response (at least to some extent) in order to be perceived as an $H$ type. In a separating equilibrium, an $H$-type's best response effort level $e_H$ must ensure that: (a) an $H$ type would (at least weakly) prefer to choose $e_H$ rather than choosing her nondistortionary best response $\rho_H^N(P^*) = t_H P^*$ and being perceived as an $L$ type, and (b) an $L$ type cannot do better by mimicking $e_H$ and being perceived as an $H$ type than by choosing $\rho_L^N(P^*) = t_L P^*$ and being perceived as an $L$ type.

Statement (a) in the preceding paragraph translates into the following inequality in $e_H$:

$$(1/2)(t_H + e_H)P^* - (1/4t_H)(e_H)^2 \geq (1/2)(t_L + t_H P^*)P^* - (1/4t_H)(t_H P^*)^2,$$

which is satisfied for all $e_H$ in the closed interval:

$$[\hat{e}^-(P^*), \hat{e}^+(P^*)] = \big[t_H P^* - (2t_H(t_H - t_L)P^*)^{1/2},\\ t_H P^* + (2t_H(t_H - t_L)P^*)^{1/2}\big].$$

Statement (b) in the preceding paragraph translates into the following inequality in $e_H$:

$$(1/2)(t_L + t_L P^*)P^* - (1/4t_L)(t_L P^*)^2 \geq (1/2)(t_H + e_H)P^* - (1/4t_L)(e_H)^2,$$

which is satisfied for all $e_H$ *not* in the open interval:

$$\big(\tilde{e}^-(P^*), \tilde{e}^+(P^*)\big) = \big(t_L P^* - (2t_L(t_H - t_L)P^*)^{1/2},\\ t_L P^* + (2t_L(t_H - t_L)P^*)^{1/2}\big).$$

Thus, the following are candidates for the $H$-type's best response (respecting the requirement of no mimicry): $e_H \in [\hat{e}^-(P^*), \tilde{e}^-(P^*)] \cup$

$[\tilde{e}^+(P^*), \hat{e}^+(P^*)]$. The interval $[\hat{e}^-(P^*), \tilde{e}^-(P^*)]$ *may* be empty, whereas the interval $[\tilde{e}^+(P^*), \hat{e}^+(P^*)]$ is always nonempty.

### Application of the Intuitive Criterion

We apply the intuitive criterion (Cho and Kreps, 1987) at the best response stage in order to facilitate the derivation of the equilibrium. This is valid because, given entrepreneur $j$'s conjectured separating strategy (equilibrium or otherwise), what remains is simply a single-entrepreneur signaling game between entrepreneur $i$ and the buyers. Moreover, if there were a separating equilibrium wherein the $H$-type partner is distorting her effort level to an extent greater than the minimum necessary to deter mimicry by her alter ego (the $L$ type), then the intuitive criterion applied at the equilibrium stage would upset it, because either $H$-type entrepreneur could reduce the extent of distortion and still be revealed as type $H$, thus improving her payoff.

According to the intuitive criterion, any of the effort levels $e_H \in [\hat{e}^-(P^*), \tilde{e}^-(P^*)] \cup [\tilde{e}^+(P^*), \hat{e}^+(P^*)]$ should be attributed to the $H$ type (because the $L$ type could never gain—even if she were to be perceived as an $H$ type—by choosing such an effort level). Thus, the $H$ type simply needs to choose her preferred value of $e_H$ from within these two intervals.

First, we argue that no effort level in $[\hat{e}^-(P^*), \tilde{e}^-(P^*)]$ will be chosen by the $H$ type. For some parameter values, this interval is empty, and therefore irrelevant. Now suppose that this interval is nonempty; because it is entirely below the $H$-type's nondistortionary best-response function $\rho_H^N(P^*) = t_H P^*$, the best candidate in this interval is its highest element, $\tilde{e}^-(P^*)$. However, straightforward calculation shows that the $H$ type would prefer the effort level $\tilde{e}^+(P^*)$ to the effort level $\tilde{e}^-(P^*)$; that is, $u_i(\tilde{e}^-(P^*), t_H, t_H \mid P^*) < u_i(\tilde{e}^+(P^*), t_H, t_H \mid P^*)$.

Thus, we can confine our search for the $H$-type's best response effort level to the interval $[\tilde{e}^+(P^*), \hat{e}^+(P^*)]$. For some parameter values, this interval lies entirely above the $H$-type's nondistortionary best-response function $\rho_H^N(P^*) = t_H P^*$, and for other parameter values, $\rho_H^N(P^*) = t_H P^*$ lies within the interval (in which case the $H$ type can reveal her type without distorting her best response to deter mimicry). Therefore, if $\rho_H^N(P^*) \geq \tilde{e}^+(P^*)$, then the $H$-type's best-response function is given by $\rho_H^N(P^*)$. On the other hand, if $\rho_H^N(P^*) < \tilde{e}^+(P^*)$, then the $H$ type will distort her effort level to the minimum extent necessary to deter mimicry by the $L$ type; that is, the $H$-type's best response will be $\rho_H^D(P^*) \equiv \tilde{e}^+(P^*)$; we will refer to this as the $H$-type's "distortionary best-response function." We conclude that the $H$-type's best response effort level (written as a function of $P^*$) is given by

$BR_H(P^*) \equiv \max\{t_H P^*, \tilde{e}^+(P^*)\}$. Comparing these two functions yields the following characterization of the $H$-type's best-response function (which is continuous at the point of transition):

$$BR_H(P^*) = \begin{cases} \rho_H^D(P^*) & \text{when } P^* < 2t_L/(t_H - t_L) \\ \rho_{\mathrm{H}}^{\mathrm{N}}(P^*) & \text{when } P^* \geq 2t_L/(t_H - t_L) \end{cases}.$$

The $L$-type's best-response function is $BR_L(P^*) = \rho_L^N(P^*)$ for all $P^*$.

## Finding $P^*$ and Showing That It Is Unique and Associating the Proper Beliefs

In a symmetric separating PBE, each entrepreneur's expected productivity must equal that of the other entrepreneur, $P^*$. Given the best-response functions derived above, the entrepreneur's expected productivity is given by $\varphi(P^*) \equiv \max\{\varphi^D(P^*), \varphi^N(P^*)\}$, where $\varphi^D(P^*) \equiv \lambda(t_H + \rho_H^D(P^*)) + (1-\lambda)(t_L + \rho_L^N(P^*)) = \bar{t} + t_L P^* + \lambda[2t_L(t_H - t_L)P^*]^{1/2}$ and $\varphi^N(P^*) \equiv \lambda(t_H + \rho_H^N(P^*)) + (1-\lambda)(t_L + \rho_L^N(P^*)) = \bar{t} + \bar{t}P^*$. Again, the transition occurs at $P^* = 2t_L/(t_H - t_L)$, so that:

$$\varphi(P^*) = \begin{cases} \bar{t} + t_L P^* + \lambda[2t_L(t_H - t_L)P^*]^{1/2} & \text{when } P^* < 2t_L/(t_H - t_L) \\ \bar{t} + \bar{t}P^* & \text{when } P^* \geq 2t_L/(t_H - t_L) \end{cases}.$$

Any value of $P^*$ such that $P^* = \varphi(P^*)$ will generate symmetric separating PBE effort levels by appropriate substitution into the functions $BR_H(P^*)$ and $BR_L(P^*)$. We will show below that there is a unique solution to the equation $P^* = \varphi(P^*)$. For some parameter values, it will be given by $P^* = \varphi^D(P^*)$. We will denote this solution by $P^D$, with the implied effort levels being denoted $e_L^D = t_L P^D$ and $e_H^D = t_L P^D + [2t_L(t_H - t_L)P^D]^{1/2}$, and we will refer to this as a "distortionary perfect Bayesian equilibrium (DPBE)." For other parameters, the solution will be given by $P^* = \varphi^N(P^*)$. We will denote this solution by $P^N$, with the implied effort levels being denoted $e_L^N = t_L P^N$ and $e_H^N = t_H P^N$, and we will refer to this as a "non-distortionary perfect Bayesian equilibrium (NPBE)."

**Lemma 1:**

(a) *There exists a unique* $P^D \in (0, \infty)$ *such that* $\varphi^D(P)(> = <)P$ *as* $P(< = >)P^D$.
(b) *There exists a unique* $P^N \in (0, \infty)$ *such that* $\varphi^N(P)(> = <)P$ *as* $P(< = >)P^N$.

*Proof of Lemma 1*

(a) Because $\varphi^D(0) > 0$ and $\varphi^D(P)$ is a strictly increasing and concave function with $\lim_{P\to\infty} \varphi^{D\prime}(P) = t_L < 1$, it follows that the function $\varphi^D(P)$ begins above the 45-degree line and crosses it just once.

(b) Because $\varphi^N(0) > 0$ and $\varphi^N(P)$ is linear in $P$ with slope $\bar{t} < 1$, it follows that the function $\varphi^N(P)$ begins above the 45-degree line and crosses it just once. □

The value of $P^N$ is easily computed to be $P^N = \bar{t}/(1-\bar{t})$. The computation of $P^D$ is more complicated, as it is given by $P^D = \bar{t} + t_L P^D + \lambda[2t_L(t_H - t_L)P^D]^{1/2}$. Let $Y \equiv [2t_L(t_H - t_L)P^D]^{1/2}$. Then $P^D = \bar{t} + t_L P^D + \lambda Y$ and $P^D = Y^2/2t_L(t_H - t_L)$. Equating these gives a quadratic in $Y$ that has one positive and one negative root. Because $Y$ is a square root (and therefore must be positive), it is the positive root that we seek. This root is $Y = \lambda t_L(t_H - t_L)/(1-t_L) + \{[\lambda t_L(t_H - t_L)/(1-t_L)]^2 + 2\bar{t}t_L(t_H - t_L)/(1-t_L)\}^{1/2}$. Substituting back into the equation $P^D = \bar{t} + t_L P^D + \lambda Y$ and solving yields the value of $P^D$.

$$P^D = \bar{t}/(1-t_L) + [\lambda/(1-t_L)]\Big[\lambda t_L(t_H - t_L)/(1-t_L) + \Big\{[\lambda t_L(t_H - t_L)/(1-t_L)]^2 + 2\bar{t}t_L(t_H - t_L)(1-t_L)\Big\}^{1/2}\Big].$$

Because $\varphi(P^*) = \max\{\varphi^D(P^*), \varphi^N(P^*)\}$, the values $P^D$ and $P^N$ provide *candidates* for equilibrium values of $P^*$. However, only one of these will *actually* provide an equilibrium value of $P^*$, depending on the prevailing parameters.

In what follows, we abbreviate symmetric separating perfect Bayesian equilibrium as SSPBE.

**LEMMA 2:** *There exists a unique $P^* \in (0, \infty)$ such that $\varphi(P)$ $(> = <)$ $P$ as $P$ $(< = >)$ $P^*$.*

*(a) If $P^N = \bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L)$, then $P^* = P^D$, and the SSPBE is distortionary.*

*(b) If $P^N = \bar{t}/(1-\bar{t}) > 2t_L/(t_H - t_L)$, then $P^* = P^N$, and the SSPBE is nondistortionary.*

*(c) If $P^N = \bar{t}/(1-\bar{t}) = 2t_L/(t_H - t_L)$, then $P^* = P^N = P^D$, and the SSPBE is nondistortionary because the distortion associated with $P^D$ is zero.*

*Proof of Lemma 2.* First note that $\varphi(0) = \varphi^D(0) = \varphi^N(0) > 0$, so $P = 0$ is never an equilibrium value. The remainder of the proof considers $P \in (0, \infty)$.

(a) Suppose that $P^N = \bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L)$. For all $P \in (0, P^N)$, $\varphi(P) = \varphi^D(P) > \varphi^N(P) > P$. Moreover, $\varphi(P^N) = \varphi^D(P^N) > \varphi^N(P^N) = P^N$. For all $P \in (P^N, 2t_L/(t_H - t_L))$, $\varphi(P) = \varphi^D(P) > \varphi^N(P)$ but $\varphi^N(P) < P$. At $P = 2t_L/(t_H - t_L)$, $\varphi(P) = \varphi^D(P) = \varphi^N(P) < P$. Because we have shown that $\varphi(P^N) > P^N$ and $\varphi(2t_L/(t_H - t_L)) < 2t_L/(t_H - t_L)$, and because $\varphi(\bullet)$ is monotonically increasing, it follows that there exists a unique $P^* \in (P^N, 2t_L/(t_H - t_L))$ such that $\varphi(P)$ $(> = <)$ $P$ as $P$ $(< = >)$ $P^*$. Because $\varphi(P) = \varphi^D(P)$ on this interval, $P^*$ is given by $P^D$. Finally, for all $P \in (2t_L/(t_H - t_L), \infty)$, $\varphi(P) = \varphi^N(P) < P$. Thus, there is only one value of $P^*$ for which $\varphi(P^*) = P^*$, and that is $P^* = P^D$.

(b) Suppose that $P^N = \bar{t}/(1-\bar{t}) > 2t_L/(t_H - t_L)$. For all $P \in (0, 2t_L/(t_H - t_L))$, $\varphi(P) = \varphi^D(P) > \varphi^N(P) > P$. For all $P \in [2t_L/(t_H - t_L), \infty)$, $\varphi(P) = \varphi^N(P)$ $(> = <)$ $P$ as $P$ $(< = >)$ $P^N$. Thus, there is only one value of $P^*$ for which $\varphi(P^*) = P^*$, and that is $P^* = P^N$.

(c) Suppose that $P^N = \bar{t}/(1-\bar{t}) = 2t_L/(t_H - t_L)$. Then it is straightforward to show that $P^D = P^N$. For $P \in (0, P^N)$, $\varphi(P) = \varphi^D(P) > P$; at $P = P^N$, $\varphi(P^N) = \varphi^D(P^N) = \varphi^N(P^N) = P^N$; and for $P \in (P^N, \infty)$, $\varphi(P) = \varphi^N(P) < P$. Thus, there is only one value of $P^*$ for which $\varphi(P^*) = P^*$, and that is $P^* = P^N$. □

Finally, the SSPBE effort levels depend on the parameter regime. A DPBE arises if $P^N = \bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L)$. In this case, $P^* = P^D$ and the effort levels are $e_H^* = e_H^D = t_L P^D + [2t_L(t_H - t_L)P^D]^{1/2}$ and $e_L^* = e_L^D = t_L P^D$. An NPBE arises if $P^N = \bar{t}/(1-\bar{t}) \geq 2t_L/(t_H - t_L)$. In this case, $P^* = P^N$ and the effort levels are $e_H^* = e_H^N = t_H P^N$ and $e_L^* = e_L^N = t_L P^N$. The beliefs that support the equilibrium are as follows: if $e \geq e_H^*$, then the buyers infer that the entrepreneur is an $H$ type; and if $e < e_{\mathrm{H}}^*$, then the buyers infer that the entrepreneur is an $L$ type. □

## Statement of Interim and *Ex Ante* Expected Payoffs Under Incomplete Information

Interim equilibrium payoffs are

$$U_H^{\mathrm{II}} = \lambda u_i\left(e_H^*, t_H, t_H \mid e_H^*, t_H\right) + (1-\lambda)u_i\left(e_H^*, t_H, t_H \mid e_L^*, t_L\right)$$
$$= u_i\left(e_H^*, t_H, t_H \mid P^*\right) = (t_H + e_H^*)P^* - \left(e_H^*\right)^2/(4t_H),$$

and

$$U_L^{\mathrm{II}} = \lambda u_i\left(e_L^*, t_L, t_L \mid e_H^*, t_H\right) + (1-\lambda)u_i\left(e_L^*, t_L, t_L \mid e_L^*, t_L\right)$$
$$= u_i\left(e_L^*, t_L, t_L \mid P^*\right) = \left(t_L + e_L^*\right)P^* - \left(e_L^*\right)^2/(4t_L).$$

If $\bar{t}/(1-\bar{t}) < 2t_L/(t_H - t_L)$, then $P^* = P^{\mathrm{D}} = \bar{t}/(1-t_L) + [\lambda/(1-t_L)][\lambda t_L(t_H - t_L)/(1-t_L) + \{[\lambda t_L(t_H - t_L)/(1-t_L)]^2 + 2\bar{t}t_L(t_H - t_L)/(1-t_L)\}^{1/2}]$, $e_H^* = e_H^D = t_L P^D + [2t_L(t_H - t_L)P^D]^{1/2}$, and $e_L^* = e_L^D = t_L P^D$. If $\bar{t}/(1-\bar{t}) \geq 2t_L/(t_H - t_L)$, then $P^* = P^N = \bar{t}/(1-\bar{t})$, $e_H^* = e_H^N = t_H P^N$, and $e_L^* = e_L^N = t_L P^N$. Thus, the *ex ante* expected payoff to a partner under incomplete information, denoted as $U^{\mathrm{II}}$, is

$$U^{\mathrm{II}} \equiv \lambda\left[(t_H + e_H^*)P^* - (e_H^*)^2/(4t_H)\right] + (1-\lambda)\left[(t_L + e_L^*)P^* - (e_L^*)^2/(4t_L)\right].$$

*Proof of Proposition 3*

(i) $U_H^{\text{II}} \geq \max_e(t_L + e)P^*/2 - (e)^2/(4t_H) > \max_e(t_L + e)P^*/2 - (e)^2/(4t_L) = U_L^{\text{II}}$, where the first (weak) inequality follows from the incentive compatibility constraint (6a); the second (strong) inequality follows from the fact that $t_L < t_H$; and the equality follows from the fact that the $L$ type plays according to her nondistortionary best-response function.

(ii) First consider $U_L^{\text{II}} = \max_e(t_L + e)P^*/2 - (e)^2/(4t_L)$, where $P^* = P^{\text{N}}$ or $P^* = P^D$ as appropriate. Note that both $P^N$ and $P^D$ are increasing functions of $\lambda$. Because the $L$ type always plays according to her nondistortionary best-response function, it follows from the envelope theorem that $\partial U_L^{\text{II}}/\partial\lambda = (t_L + e_L^*)(\partial P^*/\partial\lambda)/2 > 0$. Now consider $U_H^{\text{II}}$; the same envelope theorem argument applies when the SSPBE is nondistortionary because, in that case, $U_H^{\text{II}} = \max_e(t_H + e)P^N/2 - (e)^2/(4t_H)$. The argument is more complex when $U_H^{\text{II}} = (t_H + e_H^D)P^D/2 - (e_H^D)^2/(4t_H)$. In this case,

$$\begin{aligned}\partial U_H^{II}/\partial\lambda &= \left(P^D - e_H^D/t_H\right)\left(\partial e_H^D/\partial\lambda\right)/2 + \left(t_H + e_H^D\right)\left(\partial P^D/\partial\lambda\right)/2 \\ &= \left[\left(\partial P^D/\partial\lambda\right)/2\right]\left[t_H + e_H^D + \left\{(t_H - t_L)P^D\right.\right. \\ &\quad \left.\left. - \left[2t_L(t_H - t_L)P^D\right]^{1/2}\right\}\left\{t_L + \left[t_L(t_H - t_L)/2P^D\right]^{1/2}\right\}/t_H\right].\end{aligned}$$

A sufficient condition for this expression to be positive is that $t_H + e_H^D + \{-[2t_L(t_H - t_L)P^D]^{1/2}\}\ \{t_L + [t_L(t_H - t_L)/2P^D]^{1/2}\}/t_H > 0$; that is, if $\{e_H^D - (t_L/t_H)[2t_L(t_H - t_L)P^D]^{1/2}\} + \{t_H - (t_L/t_H)(t_H - t_L)\} > 0$. But both of the expressions in braces are positive, so $\partial U_{\text{L}}^{\text{II}}/\partial\lambda > 0$ when the SSPBE is distortionary.

(iii) Because $U^{\text{II}} = \lambda U_H^{\text{II}} + (1 - \lambda)U_L^{\text{II}}$, it follows that

$$\partial U^{\text{II}}/\partial\lambda = \lambda\left(\partial U_H^{\text{II}}/\partial\lambda\right) + (1 - \lambda)\left(\partial U_L^{\text{II}}/\partial\lambda\right) + \left(U_H^{\text{II}} - U_L^{\text{II}}\right).$$

Every term in this expression is positive by (i) and (ii), and thus $\partial U^{\text{II}}/\partial\lambda > 0$. □

*Proof of Proposition 5*

(i) Recall that $e_{rs} = t_r t_s(1 + t_r)/(1 - t_r t_s)$, $r, s = H, L$. The proof is trivial and therefore omitted.

(ii) Recall that $N(t_H, t_L) = [\Lambda(t_H, t_L), 1)$, and suppose that $N(t_H, t_L)$ is nonempty; that is, $\Lambda(t_H, t_L) < 1$. (a) Note that $e_{HH} > e_H^N$ if and only if $t_H^2/(1 - t_H) > t_H\bar{t}/(1 - \bar{t})$; this inequality holds for all $\lambda \in (0, 1)$, and thus for all $\lambda \in N(t_H, t_L)$. Because $e_H^N > e_{HL}$ if and only if $t_H\bar{t}/(1 - \bar{t}) > t_H t_L(1 + t_H)/(1 - t_H t_L)$, it follows (after some algebra) that $e_H^N > e_{HL}$ if and only if $\lambda > t_L/(1 + t_L)$. Because $t_L/(1 + t_L) < \Lambda(t_H, t_L)$, all $\lambda \in N(t_H, t_L)$ exceed $t_L/(1 + t_L)$ and thus $e_H^N > e_{HL}$ $\forall\lambda \in$

$N(t_H, t_L)$. (b) Note that $e_L^N > e_{LL}$ if and only if $t_L\bar{t}/(1-\bar{t}) > t_L^2/(1-t_L)$; this inequality holds for all $\lambda \in (0, 1)$, and thus for all $\lambda \in N(t_H, t_L)$. (c) Because $e_L^N$ $(> = <)e_{LH}$ as $t_L\bar{t}/(1-\bar{t})(> = <)t_L t_H(1+t_L)/(1-t_H t_L)$, it follows (after some algebra) that $e_L^N$ $(> = <)e_{LH}$ as $\lambda$ $(> = <)$ $1/(1{+}t_H)$. In particular, $e_L^N \geq e_{LH}$ for $\lambda \in [\max\{\Lambda\ (t_H,\ t_L),\ 1/(1+t_H))\},\ 1)$ (this interval is always nonempty) and $e_L^N < e_{LH}$ for $\lambda \in [\Lambda\ (t_H, t_L), 1/(1+t_H))$ (this interval may be empty).

Let us review several results that will be used in the proof of parts (iii) and (iv). First, at any point $(t_H, t_L)$ such that $\Lambda(t_H, t_L) > 1$, the SSPBE will be distortionary for all $\lambda \in (0, 1)$. Second, because the equation $t_L = t_H^2/(2-t_H)$ defines the locus $\Lambda(t_H, t_L) = 1$ and because $\Lambda(t_H,\ t_L)$ is increasing in $t_L$ for fixed $t_H$, the fact that $\Lambda(t_H,\ t_L) > 1$ implies that $t_L > t_H^2/(2-t_H)$. Third, $P^* = P^D > P^N$ for all $\lambda \in (0, 1)$ (this inequality was obtained in the proof of Lemma 2).

(iii) Note that $e_L^D > e_{LL}$ if and only if $h(\lambda) \equiv t_L P^D - t_L^2/(1-t_L) > 0$. But because $P^D > \bar{t}/(1-t_L)$, it follows that $h(\lambda) > t_L\bar{t}/(1-t_L) - t_L^2/(1-t_L) > 0$ for all $\lambda \in (0, 1)$ and thus for all $\lambda \in D(t_H, t_L)$.

(iv) Note that $e_H^D > e_{HH}$ if and only if $g(\lambda) \equiv t_L P^D + [2t_L(t_H - t_L)P^D]^{1/2} - t_H^2/(1-t_H) > 0$. Note further that $\lim_{\lambda\to 0} g(\lambda) < 0$ but $g'(\lambda) > 0$. We will show that $\lim_{\lambda\to 1} g(\lambda) > 0$. Because $g(\lambda) > t_L P^N + [2t_L(t_H - t_L)P^N]^{1/2} - t_H^2/(1-t_H)$ for all $\lambda \in (0, 1)$, $\lim_{\lambda\to 1} g(\lambda) \geq \lim_{\lambda\to 1} t_L P^N + [2t_L(t_H - t_L)P^N]^{1/2} - t_H^2/(1-t_H) = t_L t_H/(1-t_H) + [2t_L(t_H - t_L)t_H/(1-t_H)]^{1/2} - t_H^2/(1-t_H) > 0$. (This last inequality follows from the fact that $t_L > t_H^2/(2-t_H)$). Finally, note that $e_L^D > e_{LH}$ if and only if $k(\lambda) \equiv t_L P^D - t_L t_H(1+t_L)/(1-t_H t_L) = [t_L/(1-t_L)][\bar{t} + \lambda Y] - t_L t_H(1+t_L)/(1-t_H t_L) > 0$, where $Y$ is as defined above in the construction of the equilibrium. Let $Y(1) \equiv \lim_{\lambda\to 1} Y(\lambda)$, and notice that $Y(1) > t_L(t_H - t_L)/(1-t_L)$. $\lim_{\lambda\to 0} k(\lambda) < 0$ but $k'(\lambda) > 0$. We will show that $\lim_{\lambda\to 1} k(\lambda) > 0$. We can write $\lim_{\lambda\to 1} k(\lambda) = [t_L/(1-t_L)][t_H + Y(1)] - t_L t_H(1+t_L)/(1-t_H t_L) > [t_L/(1-t_L)][t_H + t_L(t_H - t_L)/(1-t_L)] - t_L t_H(1+t_L)/(1-t_H t_L) > 0$ (following some algebra). □

## Supporting Information

Supplementary information is available online as the Supporting Appendix; http://www.vanderbilt.edu/Econ/faculty/Daughety/SuppAppforHiddenTalents.pdf

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