Economic models

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1 Learning Objectives

After mastering the material in this chapter, the reader should:

1. understand that an economic model is "a logical representation of whatever a priori or theoretical knowledge economic analysis suggests is most relevant for treating a particular problem."
2. understand why economists make such pervasive use of models;
3. know that the key components of models are equations that represent assumed logical interrelationships among variables;
4. know the distinction between endogenous and exogenous variables;
5. know what it means to "solve a model;"
6. understand how "curve - shifting" conveys the changes in a model's solution that arise from changes in exogenous variables;
7. understand the difference between general functional notation and a specific functional form, and understand why economists use specific functional forms;
8. understand the difference between a structural model and its reduced form.

2 Introduction

International economics as usually taught centers around a string of models: the Ricardian Model, the Specific and Mobile Factors Model, the Heckscher - Ohlin Model, the model of monopolistic competition, and a variety of others, depending on which text one uses. What is difficult for students is that these models are not (and are not usually presented as) more and more elaborate treatments of the same basic model, but rather are in some ways fundamentally different. Different models are used to understand different phenomenon. Understanding all these different models may appear a daunting task. What helps in this task is an understanding of the generic structure that underlies virtually every economic model. Such an understanding provides the mental hooks on which to hang the new features of different models. This chapter will provide this understanding.

We start with illustrations of the power and pervasiveness of models even outside of economics. We then provide an outline of the components of a generic economic model and a discussion of how economists use their models. We then illustrate these general concepts by building a model to address an international economics policy issue that arose in 2004.

3 The power and ubiquitousness of models

Consider the following problem: At exactly 6:00 AM, a monk leaves the base of a mountain to begin climbing the path to the top. He arrives at the top at 6:00 PM, where he spends the night in the monastery. The next morning at 6:00 AM, he begins descending along the same trail that he used to climb the mountain. At 6:00 PM he arrives back at the base of the mountain.

The question we pose is the following: Must he have been at some identical point on the trail at the exact same time of day on both his trip up and his trip down?

Take a few minutes to think about this problem before reading further and finding the answer. Be clear about what the problem asks: is there a spot on the trail, say, for example, at length 300 meters from the bottom, that the monk reaches at the same time of day, for example 11:30 AM, both on his way up and on his way down? You are not told whether or not the monk travels at a constant rate of speed or whether he rests sporadically during his journey. All you know is his departure and arrival times and that he travels the same trail. You are not asked to describe a particular time and place, only to decide whether there must be such a time and place.

This is a hard problem. When students are broken into a dozen or so small groups of four or five people and given ten minutes or so to work on this problem, most of the groups cannot decide if the answer is true or false. A few groups conclude it is false because it seems improbable to them. A few groups answer a slightly different question than the one asked. They make the following conditional claim: "If the monk travels at the same rate both up and

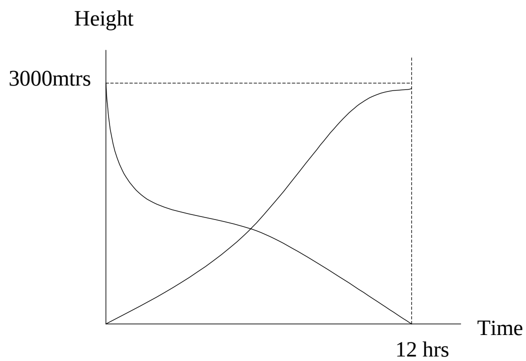


Figure 1: The Monk's paths

down the mountain, he will be at the midpoint of the trail at noon on both days." They stop at this point because any enrichment of possibilities, e.g., the monk travels faster downhill for the first six hours, and then rests for awhile, makes it too difficult for them to figure out what might happen. Usually, only one or two groups conclude it is true. This is the correct answer.

How did the few groups who got the right answer arrive at their conclusion? Invariably they did so in one of two ways. The most frequent method was where the group (or someone in the group) hit upon the idea of drawing a graph of the monk's progress. On the vertical axis they measured the length of the trail from a starting point at the bottom of the mountain, and on the horizontal axis they measured time from a starting point of 6:00 A.M. They then plotted where he is on the trail at every moment of his twelve-hour trip up the mountain. Then they plotted where he is on the trail at every moment of his twelve-hour trip down the mountain. If you try this, you will see that you cannot plot these paths without them crossing. The crossing point represents a point where the monk is at the same point on the path at the same time of day. A sample graph is depicted in Figure 1. The figure illustrates the progress of a monk who travels at varied speeds to emphasize that the answer to the question does not require specific information on the exact speed of the monk.

When this solution is presented to the other groups, most students immediately see that this is a correct analysis. However, a few students sometimes remain unconvinced. For them, the other method used to uncover the correct answer is the one that is persuasive.

This method is an argument by analogy. Invariably some group poses the following "thought experiment": Suppose there are two monks, one at the top of the mountain, one at the bottom. They both start to travel at 6:00 AM, one going down and one going up, and they both must finish their trip by 6:00 PM. Clearly they must meet on the path. If one thinks of the monk coming down the mountain as mimicking the pace of the lone monk of the first problem as he (the lone monk) descends on the second day, then the point where the two monks meet satisfies the requirement that the lone monk be at the same spot at the same time both on the way up and on the way down.

What this group has seen is that the two-monk thought experiment is essentially the same as the original problem. It is analogous or alike "enough" to the first problem, so we see that they capture the same phenomenon.

With both successful approaches, what students are doing to solve this problem is constructing a *model*: a logical representation of the essence of the situation. Note that this model is not an exact re - creation of the problem: neither a graph or a two-monk thought experiment is *exactly* like the original problem. In fact, one can (with great difficulty) imagine a true skeptic who is not convinced by either of these models. Such a skeptic might want to follow the monk up and down the mountain, measuring distance and recording times. Of course, even then, all the skeptic has proven is that one particular monk on two particular days has been on the identical spot on the trail at the same time. To understand anything useful, that is, something that applies to other similar experiences, the skeptic still must abstract.

The point here is that to solve problems people construct abstract models. What a model does is eliminate insignificant or inconsequential detail, leaving the essence of the problem exposed for analysis. When economists tackle problems, this is what they must also do.

Models are not only essential for solving hard problems, they are also ubiquitous in everyday life. Consider what is meant by "a straight line". Most people remember the definition they learned in high school geometry: a straight line is the shortest distance between two points. They also generally agree that in everyday usage, many objects, such as the edge of a door, are straight lines. Upon reflection, though, everyone realizes that no object in the world satisfies the geometric definition: under a powerful enough microscope, every straight line will appear curved and jagged. We must conclude, then, that straight lines in the real world are to us like pornography was to Justice Potter Stewart of the U.S. Supreme Court: we can't define it, but we know it when we see it.

The conclusion we can draw from this discussion is that any attempt to make sense of the world involves abstraction and simplification. To make progress, we are forced to do things like some people do to solve the monk problem: "*Assume* there are two monks".

The reader may have heard the old joke about the shipwreck that left a chemist, a physicist, and an economist stranded on a deserted island. The only thing saved from the ship was a box of canned food. The three survivors' problem was how to open the cans. The chemist suggested they gather some roots and vegetation from the island from which he would make an acidic solution that they could use to eat through the top of the cans. The physicist suggested they use his eyeglass lens to focus sunlight on the cans and melt the top. The economist chuckled at the complexity of these two solutions, and then offered his own solution: assume a can-opener.

Even economists, despite being dismal scientists, can laugh at this joke. They know that they make many assumptions that appear heroic in their distance from what appears to be the real world. Again and again, students and non-economists express amazement at the plethora of assumptions used by economists, assumptions like: firms maximize profits, households choose a most-preferred bundle of commodities, and so on. The question one must keep in mind is not whether these assumptions exactly capture reality, but rather whether they capture the essence of the problem at hand, and thus help make progress in understanding or solving some economic problem.

In contrast to many other social science disciplines, economics has a rather well-developed concept of what are the key components of their models. These key components form a generic structure within which virtually all economic models fit. Once one has learned this generic structure, he or she can better remember and use the more specific models designed to address specific problems. For this reason, we are going to devote considerable attention to this generic structure. In the remainder of this chapter, we will develop this generic structure and illustrate it with concrete examples of a model with which the student might have some familiarity from a principles course.

4 The structure, language, and depiction of economic models.

4.1 Some definitions

We noted above that an economic model is a "logical representation of the essence of the situation." A more complete definition of an economic model is provided in Kane (1968):

> An economic model is a logical (usually mathematical) representation of whatever a priori or theoretical knowledge economic analysis suggests is most relevant for treating a particular problem.

a priori: related to or derived by reasoning from self -evident propositions; being without examination or analysis.

What do we mean by a "logical representation?" For virtually every economic model, such a representation can be boiled down to a collection of equations that express interrelationships among variables. How do economists come up with these collections of equations, and how do they use them to address issues? In this section, we will explicitly delineate the "thought protocols" used by economists as they construct and use models. In doing so, we provide a more detailed description of the variables and equations that constitute elements of models, and a detailed description of how economists use these models. As we do this, we also introduce some basic terminology and review the key concepts from analytic geometry that are most useful to economists.

To orient you as we develop this detailed description, we first pose an international economic problem that is typical of those analyzed by economists.

4.2 A problem: The effects of an end of EU sugar - production subsidies

In 2004, the European Union's agricultural minister stated that subsidies paid to European Union sugar producers were going to removed at some time in the future. Many people in countries such as Brazil applauded this announcement, anticipating that sugar producers in their countries would be better off if the subsidies were removed.

This is the sort of problem economists frequently analyze: what are the effects of a contemplated policy change on the economic well-being of various participants in the economy? We can illustrate how economists analyze a problem like this using the basic supply/demand model for which the basics are familiar to anyone with a passing exposure to economic analysis.

4.3 Elements of models

4.3.1 Variables

First, for the problem at hand, economists think about what variables are likely important and what variables are likely unimportant. For example, when thinking about what determines an individual's consumption of sugar per year (itself a variable), economists believe that variables like the price of sugar, the price of related goods, and disposable income per year are important. In contrast, they don't believe variables like hair color, height, phase of the moon, or gender are important. They may be important for the study of another problem, such as the proportion of expenditure spent on clothing, but not for the problem of explaining total sugar purchases. An economist makes this choice of what variables are important and what variables are inconsequential on the basis of a variety of things, including intuition, introspection, information gleaned from related disciplines such as psychology or sociology, and from empirical observations. This choice, never irrevocable, represents the first part of the art of modeling.

A **variable** is a quantity free to take on any of a number of permissible values.

An important feature of variables used in economic modeling is that they can all be represented by numbers. For some variables, this seems obvious: prices and quantities, for example, are naturally described as numbers. For other variables, though, assigning numbers to them is not an obvious step, but can be thought of as just a way of renaming things. Even things like phases of the moon or hair coloring can be assigned different numerical values. For example, if hair color was thought to be important for some model, red could be assigned (i.e., named) the number one (1), brown the number two (2), purple the number three (3), and so on. Assigning variables values that are numbers allows economists to use the techniques of analytic geometry to depict these variables and their interactions with each other.

4.3.2 Logical structuring and representation

After making this first-stage decision about what variables are important, economists then make a logical structuring and representation of what they think are the interrelationships between these variables. There are two key parts to this process.

Endogenous versus exogenous variables First, economists make a further distinction among variables: they dichotomize them into **endogenous** and **exogenous**. An exogenous variable is defined as having a value determined outside the model. That is, the value of an exogenous variable is taken by the economist as "god-given" and not to be determined by the economist. An exogenous variable is sometimes referred to as an independent variable. An endogenous variable, on the other hand, is defined as having its value determined jointly by the particular values taken by the exogenous variables and by the logical relationships among variables within the model. Endogenous variables are sometimes referred to as dependent variables.

Exogenous variable: having a value determined outside the model.

Endogenous variable: having a value jointly determined by the values of exogenous variables and by the logical relationships among variables in the model.

Equations and other statements Once a decision is made about what variables are important and, of these, which variables are endogenous and which are exogenous, then the interactions between these variables must be specified. Usually these interactions are represented as a system of equations that consist of definitions, identities (things true by definition), technical descriptions, behavioral hypotheses, and equilibrium conditions. These relationships consist of statements like: "Consumption per year is an increasing function of disposable income per year", or "the quantity supplied per unit of time is an increasing function of the own price of the good".

There are a number of ways to depict a relationship between variables. Before describing these, let us be clear about what we mean by "relationship." When, for example, we state that "quantity supplied per unit of time is an increasing function of the own price" we are asserting a *systematic* relationship between two variables: price, and quantity per unit of time. For any specific relationship, there are three ways to represent it: with a table of values, with a mathematical expression, and with a graph. A table of values is the most concrete way to represent such a relationship, but also the most cumbersome. A mathematical expression uses symbols to represent variables and algebraic operations to describe the relationships between these variables, and is both economical and flexible. Unfortunately, these very qualities make them more abstract and intimidating to many people.

Fortunately, because variables take on values that are numbers, many equations can be depicted as graphs. For many people, a visual depiction of mathematical statements provides greater understanding of the underlying logical relationships. This observation accounts for the long-standing appeal of analytic geometry as a tool for helping people understand abstract mathematics. Most of the models used in undergraduate economics are simplified so as to permit their representation and manipulation with the tools of analytic geometry.

As we work through our specific example in this chapter, we will review those tools most useful to us.

There are also other statements that involve variables and represent other aspects of a model but do not initially take the form of equations. Because economics is often concerned with choices made by individuals, firms, governments, and organizations, economic models frequently include statements that are descriptions of an action taken by an economic agent such as:

> "Individual i chooses the most-preferred bundle of commodities that lie within her or his feasible set."

This description of **optimizing behavior**, which represents a behavioral assumption that an individual (individual i) chooses a bundle of things from a collection of bundles that is within her or his budget ("within her or his feasible set") so as to make her or him self as well-off as possible. From other economic texts or classes you may have heard this described as an assumption that someone **maximizes** his or her *"utility"* by choice of a particular bundle of commodities from those bundles he or she could afford.

Other descriptions of actions might be:

> "Firm j chooses the level of employment that maximizes its profits."

This instruction represents the assumption that firms choose levels of inputs into a production process so as to maximize profits.

Such actions, when followed, lead to **decision rules** that take the form of behavioral equations. In particular, they relate an endogenous variable (also known in this context as a *choice variable*) to what are treated as exogenous variables from the point of view of the economic agent that does the choosing. Perhaps the most familiar decision rule is an individual's **demand function**: it describes the choice by an individual of a quantity consumed, where the quantity chosen is determined by the values of variables (taken as exogenous by the individual) such as prices and income.

Parameters In some models, the equational statements involve variable-like entities called *parameters*. Parameters are like exogenous variables in that they represent a number, e.g., "six," and in that their values are given exogenously, that is, from outside the model. We distinguish between parameters and exogenous variables because we frequently think of parameters as representing aspects of an economy that change less frequently than do those aspects represented by exogenous variables.

4.4 Solving the model

4.4.1 The canonical question

At this point, the model is complete. Now the economist frequently uses the model to answer the following canonical question:

Canonical: conforming to a general rule; reduced to the simplest or most clear schema possible.

> What is the relationship between the values of the endogenous variables and the values of the exogenous variables?

Answering this question is referred to as "solving the model."

The term "relationship" in the statement of the canonical question is not very precise, and needs some elaboration. For some models, usually ones that we develop as learning exercises rather than ones that we view as true working models, the form of this relationship is very exact. We want to know: What are the actual numerical values of the endogenous variables for any particular given permissible values of the exogenous variables?

For many economic models, the specific form of the canonical question asked is a slight variation of the one posed above. It asks: What are the *directional changes* in the values of the endogenous variables for given, arbitrary *directional changes* in the values of the exogenous variables? That is, instead of specifying unique numbers as values for each of the exogenous variables and asking what are the associated numerical values of the endogenous variables, it posits a numerically-unspecified directional change, i.e., increase or decrease, in the value of an exogenous variable and asks: What are the associated numerically-unspecified directional changes in some or all of the endogenous variables?

The basic reason we ask a model this question about *changes* is that our economic models don't give us enough information to make the detailed specifications necessary to answer the first question. Many of the behavioral equations that make up our models can only be specified in the weaker terminology of qualitative changes. For example, a complete model of the interactions between an assumed exogenous variable such as the Fed Funds interest rate and other endogenous macrovariables such as employment and inflation might have as one equation: "Consumption is an increasing function of disposable income." There is information here: if disposable income goes up, consumption goes up. But the information not here is the *quantitative* relationship that tell us how much consumption goes up in response to a particular numerically-valued change in disposable income.

Without quantitative information embedded in our models, the best we can hope for is that our models can answer this form of the canonical question. The ability of many economic models to map values of exogenous variables to values of endogenous variables has been compared to the ability of a Russian tank commander in the early days of World War II to predict the response of his tank to a movement in the driving controls. These tanks were allegedly crude affairs, and movement of the tank was controlled by a big lever. If the lever was pushed forward, the tank moved forward. The farther forward the lever was pushed, the faster the tank would go. If the lever was pulled back, the tank went backward. The farther back the lever was pulled, the faster in reverse the tank went. The problem was that the lever was very "stiff:" the driver had to apply a lot of pressure to get it to move at all, and sometimes it would then move a lot, and sometimes a little. Hence, the commander could always tell in which direction the tank would move - forward or backward - but not how fast. Many economic models have that same characteristic: when an exogenous variable changes, the model can tell us in what direction the endogenous variables will move, but not by how much.

Why is this general question of the relationship between endogenous and exogenous variables the *raison d'être* (reason or justification for existence) of an economic model? First, many of the real-world problems of interest to economists are problems for which policy advice is relevant and important. For example, the chairman of the Federal Reserve Board (The Federal Reserve or "Fed" is the monetary authority in the United States) may want to know what will be the likely effects on national output, the inflation rate, the unemployment rate, and the exchange rate if the Federal Reserve decides to lower the interest rate it charges for loans to commercial banks. This interest rate is something the Fed can control if it so desires, so it can be treated as exogenous to the rest of the economy. In most macroeconomic models, a change in this exogenous variable engenders changes in the endogenous variables such as output, unemployment, inflation, and the exchange rate. Hence, this canonical question that economists ask of their models is exactly the question policy-makers should be interested in.

Second, this question helps economists avoid the particularly pervasive problem of confusing correlation with causality. As an example of this problem, consider the following theory: drinking diet cola makes people overweight. The evidence in support of this theory is the observation that one sees mostly overweight people buying diet cola at the grocery store. Another "theory" about another phenomenon we could propose is that the installation of storm windows brings on winter. The evidence in support is that every year the onset of winter is preceded by many people putting up their storm windows.

These two "theories" are relatively easily dismissed despite the presence of corroborating evidence that takes the form of correlation between the hypothesized cause and the effect. The dismissal is easy because we have good theories about what is exogenous and endogenous in these two examples. In both cases, the "evidence" is a correlation between the endogenous variables. What we really believe is that other *latent* (present though not directly visible) exogenous variables are changing, which are in turn changing the values of the endogenous variables. In the "diet cola leads to obesity" theory, we believe the truth is that already-heavy people are choosing diet cola in an attempt to reduce their caloric intake and get thinner. The latent variable that causes people to show up at the diet cola section and that causes these people to be heavy is "desire to get thinner." In the storm window example, the latent variable that causes both cold weather and people to put up their storm windows is the movement of the earth around the sun.

While in these two examples the fallacy inherent in each theory is transparent, political candidates, government officials and other public intellectuals often make similar errors. For example, a recent study has shown that if parents ask their children two questions before they leave the house, the children are less likely to use drugs. The two questions are: (1) Where are you going, and (2) When will you be home? Some people (perhaps politicians) will be tempted to infer from this that a program to teach parents to ask these questions will reduce drug use by teenagers. Such a program is likely to have little effect, though, because what the study has probably found is that caring, involved, parents ask these questions, and the "latent variable" of being a caring, involved parent causes such parents to ask such questions **and** to do a variety of other things that reduce their childrens' likelihood of taking drugs.

4.4.2 Solution strategies

Sub-models Many economic models, in their entirety, involve many equations and many variables. As a strategy for understanding and explicating such a complex system, economists build sub-models that they then use to build the complete model. In these sub-models, variables (and sometimes parameters) that are ultimately and fundamentally endogenous within the model as a whole, are treated as exogenous.

Mathematics Powerful mathematical techniques are available for solving systems of equations, and an economic model can ultimately be represented as a system of equations. Advanced treatments of economics and economic research being described for other professional economists often uses these techniques. Even in these treatments, though, economists often use graphical techniques to help themselves and their readers understand the components of their models and how these components fit together. For non-economists, these graphical techniques are paramount for helping them understand a model.

Graphs A picture is worth a thousand words, and is also worth a few equations. Most people, including most economists, find graphs enormously helpful in understanding complex relationships between variables. Because variables take on values that are numbers, the techniques of analytic geometry can be used to depict economic models. For many people, though, the use of graphs is only slightly less confusing than equations in helping them understand an economic model. For them, the following graph used by P.J. O'Rourke in his humorous layman's guide to economics, *Eat the Rich*, sums up their feelings about graphs in economics.

Fortunately, there are some general features of how economists use these tools that will help people keep from getting lost in a thicket of complicated-looking graphs. First, it helps to know what visual constraints economists impose upon themselves and how these constraints shape their graphs. Second, it also helps to know the final destination of the trip when the trip takes us through a maze of graphs.

Dimensionality Economic models, especially those used in international economics, frequently have many variables and many logical interrelationships between these variables. Unfortunately, a simultaneous depiction that human beings can understand of all these variables and interactions is impossible: most of us can't envision things in more than two, and at most sometimes three,

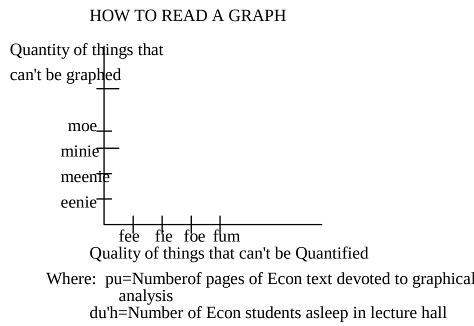


Figure 2: The O'Rourke graph

dimensions. Hence, economists try to collapse information by a variety of tricks into two-dimensional pictures. This is frequently the sole reason for some seemingly-complicated graph; it is simply a two-dimensional way-station helping us to get from one part of a model to another. It is closely related to the use of sub-models.

The canonical question and economic graphs Remember that economists are trying to use their models to answer a particular question: what is the relationship between exogenous and endogenous variables? The final graphs in a presentation of a model (or sub-model, for that matter) are designed to answer this question. The strategy, then, is to depict on a two-dimensional graph two independent relationships between at most two endogenous variables. The intersection of these relationships determines a pair of numbers that is at least part of the solution to the model. We say this pair may be just a part of the solution because in models with more than two endogenous variable, there will have to be other numbers that are part of the solution, perhaps depicted in other related two-dimensional graphs. By judicious choices, the economist hopes to be able to have only one of these relationships shift in the two-dimensional plane when an exogenous variable changes its value. This allows one to "read off" from the graph the induced change in the values of the endogenous variables.

We now move on to an in-depth description of the usual ways that economists create and express the logical interactions between the variables in a model. To make the concepts concrete and memorable, we will illustrate them with a specific model familiar to a student who has had an economic principles class: the microeconomic partial-equilibrium model of supply and demand for a particular good or service. The good in question is sugar, and the question we ask of this model is what happens to production of sugar by non - European producers if subsidies to European Union sugar producers are eliminated. We make simplifying assumptions in our model in order to help us answer this specific question.

5 The microeconomic partial-equilibrium model of competitive demand and supply for sugar

This model, parts of which are familiar from principles courses, is constructed from two sub-models: a model of demand, that is, a model of how individuals make decisions about how much of a particular good to consume over a given period of time; and a model of supply, that is, a model of how firms choose how much of the good to produce and sell during the given time period. The two sub-models are linked together by the equilibrium condition that demand equals supply. The endogenous variables for the model as a whole are the quantity of sugar per unit of time produced and consumed, and the equilibrium price. The exogenous variable is a subsidy that may be paid to European Union sugar producers. A model designed for other purposes, such as forecasting sugar production, would involve other exogenous variables such as consumer income, the prices of related goods, the price of agricultural inputs, weather, and a host of other possibilities. *For the problem at hand*, we make the simplifying assumption that these other variables don't matter.

We use the modifier *partial equilibrium* because we are looking at interactions in one market only. This means we ignore potentially important feedback effects between one market and others. We are also deferring to a later point a full development of a deeper model of consumer and producer behavior. Our primary goal here is to provide a self-conscious description of a familiar model in terms of its generic parts.

5.1 A note on notation

One of the more boring and tedious parts of learning a model is paying attention to the notation used to symbolize the various parts of the model. In particular, in models used in international economics, in which it is important to keep track of what happens to different individuals and different firms, all of which need to be identified by country location, there are a multitude of subscripts and superscripts. Unfortunately, there are few shortcuts that can help here: learning a model requires attention to a sometimes - extensive collection of symbols. The rule we follow here is to err on the side of clarity rather than simplicity.

Not always, but frequently, variables are represented by letters, e.g., x, and parameters, those exogenous-variable-like entities, by Greek letters. Greek letters are especially useful because they tend to impress the unitiated and lead them to think economics is akin to rocket science.

5.2 Demand submodel

5.2.1 Variables

First consider the demand sub-model. What are the important variables that influence the decision of how much sugar an individual consumer might wish to purchase over a given time period? Observation, introspection, and past investigations suggest that, in general, a number of variables are important: the price of sugar, the price of related goods, and the income of the individual. For the consumer, these variables are exogenous, that is, variables whose values are assumed by the consumer to be unaffected by his or her decisions. The endogenous variable in this sub-model is the quantity per unit time that is bought.

As noted, *for our purposes*, we will assume that the only variable that is important is the price of sugar. To help orient the reader to the more familiar model in which other variables are important, though, we treat an individual's income as important as well. By adding this additional variable, we also illustrate *why* we can treat it as unimportant for our question.

5.2.2 Logical structuring

Let us go through the list of exogenous variables and recount what the theory learned in principles courses tells us about the qualitative impact of hypothetical changes in the value of these variables on the quantity demanded per unit of time. For each variable, the thought experiment being conducted holds constant the values of the other exogenous variables. This is called the **ceterus paribus** assumption: ceterus paribus is a Latin phrase meaning "other things being equal".

ceterus paribus: other things being equal.

1. Own price. Ceterus paribus, as a good's own price goes up, the quantity demanded per unit of time is expected to decrease. This inverse relationship between own price and quantity demanded is sometimes know as the Law of Demand.

2. Income. Ceterus paribus, as an individual's income goes up, the quantity demanded per unit of time goes up.[1]

These two sentences describe what is frequently called an individual demand curve, or individual demand function, or individual demand schedule. Note that it describes how the endogenous (or "choice") variable is assumed to change in response to a hypothesized increase in the value of another exogenous variable, *holding constant the values of the other exogenous variable.*

[1] You may have learned that the quantity demanded per unit of time may go down if the good in question is an "inferior" good. For normal goods, an increase in income increases consumption of the good; for an inferior good, an increase in income reduces consumption of the good. For most applications, the assumption of normality is appropriate.

Functional relationship We can express the logical relationship assumed to hold between these variables in a useful shorthand method by making use of the concept of a **function** and by denoting each variable by a symbol. Denote the quantity of sugar demanded per unit of time by an individual consumer as S_i^d, (the S is mnemonic for "sugar", the superscript d reminds us this is a demand relationship, and the subscript i indicates this is for a particular individual, individual i), and income per unit of time by Y_i. The equation that embodies the information contained in the two sentences that described an individual's demand curve can be written as:

$$S_i^d = f_i(\overset{-}{P_S}, \overset{+}{Y_i}),\ i = 1, 2, ...I \tag{3.1}$$

and is to be read as: "The quantity of sugar demanded per unit of time by individual i is a decreasing function of the price of sugar, and an increasing function of income per unit of time." The variables P_S and Y_i are called the *arguments* of the function f_i. The algebraic signs ("plus" and "minus") above each argument indicates the direction in which the value of the left - hand - side variable moves in response to an increase in the value of argument, ceterus paribus. The statement that "the quantity demanded is a *function* of the variables P_S and Y_i means that once particular numerical values are specified for each of these two variables then there is a unique associated numerical value for the variable S_i^d. We will shortly elaborate on this concept.

A demand function is also referred to as a demand schedule. This terminology emphasizes that the relationship between the exogenous variables and the quantity demanded can also be expressed in the following way: A demand schedule tells how much of a good (per unit of time) would be bought at any possible values for price and income. That is, for every set of values of exogenous variables, it "reads off", much like reading information from a schedule, the quantity that would be demanded.

An example of such a schedule might look like Table 3.1. In that table, column one (1) specifies values for the endogenous variable S_i^d, and the other two columns specify values for the exogenous variables P_S and Y_i. The values for the exogenous variable Y_i is set at one (1). Three different values of the exogenous variable P_S are stipulated: one - fourth ($\frac{1}{4}$), one (1), and two (2). For each set of values for the exogenous variables, e.g., $\{P_S = 1, Y_i = 1\}$, the quantity demanded per unit of time associated with that set is specified in the first column.

S_i^d	P_S	Y_i
1	$\frac{1}{4}$	1
$\frac{1}{4}$	1	1
$\frac{1}{8}$	2	1

Table 3.1

Now, the information captured by our notion of the demand relationship could be captured in a collection of tables like Table 3.1, each table having a different set of values for exogenous variables and associated values for the endogenous variable. There are two problems with expressing the information embodied in our equational statement of demand via such a set of tables. First, the tables are too restrictive for our purposes. Our theory, for reasons that will be clearer after the next chapter, is not usually capable of predicting exact relationships between actual numbers. Rather, it is a qualitative prediction. Hence, a table of values can only serve as an example, and cannot convey the general proposition about the variables embodied in our equational statement.

Second, even as an example, a table of values is cumbersome. First of all, for a market such as the one we are describing, the units for quantities per unit of time are surely on the order of millions of kilograms per year. That is, when we stipulate "$S_i^d = 1$" we surely should interpret the number one (1) as measuring hundreds of kilograms per year. As a consequence, a table of values should have many more entries, ranging from one kilogram to one million kilograms. Second, even ignoring this problem, we would need a different table for each different set of values for the exogenous variables. Even with the values of the three variables restricted to whole numbers between zero and five, this becomes a massive number of tables.

The above table, though, can help us understand both the idea of a function and how we use the tools of analytic geometry to depict functions. Note that Table 3.1 keeps the value of Y_i constant over all three rows. The only variation is between the values of the exogenous (to the consumers) price P_S and the quantity demanded per unit of time, S_i^d. That is, when we look at only the two variables whose values change row to row, we have *a set of ordered pairs of numbers* (P_S, S_i^d) such that to each value of the first variable there corresponds a unique value of the second variable. The set of ordered pairs from our example is $\{\left(\frac{1}{4}, 1\right), (1, \frac{1}{4}), (2, \frac{1}{8})\}$. A *set* can also be thought of as a collection, and standard notation uses the curly braces $\{\}$ to identify a set, and uses parenthetical brackets () to denote members of a set. The idea of an *ordered* pair is that the first element of any pair always represents the same variable, and the second element also always represents the same (albeit different from the first variable) variable.

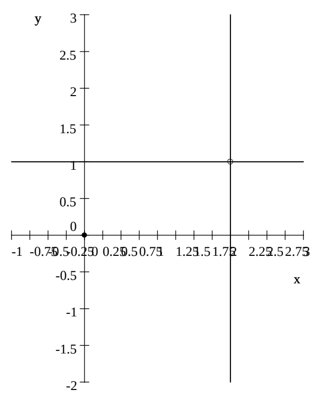
The formal mathematical definition of a function of two variables is in fact just this: a set of ordered pairs of numbers (P_S, S_i^d) such that to each value of the first variable P_S there corresponds a unique value of the second variable S_i^d. We emphasize that a two-variable function is a set of ordered pairs of numbers because analytic geometry gives us tools to depict ordered pairs of numbers, namely points (and collections of points known as geometric curves) in the Cartesian plane. For many people, this depiction is the key tool for understanding logical interactions among variables.

The key ideas of analytic geometry

Analytic geometry allows us to picture ordered pairs of numbers and algebraic equations in terms of points and geometric curves. For many people, this depiction is the key tool for understanding logical interactions among variables. The key idea, the discovery of which is credited to the French mathematician Descartes (1596-1650) involves locating a point in a plane by means of its distance from two perpendicular axis. Such a plane is known as a “Cartesian plane” and points in this plane are located by pairs of numbers known as “Cartesian coordinates.” This terminology is in commemoration of Descartes. We briefly review these concepts before developing graphical representations of the logical relationships that make up our sub-model of demand.

Coordinates The fundamental idea in analytic geometry is the establishment of a one-to-one correspondence between numbers or groups of numbers and points in a geometric space. “One-to-one” means that for every unique point there corresponds a unique pair of numbers. Of most use to undergraduate-level economics is the correspondence between points in a plane and pairs of numbers. For our partial-equilibrium demand-supply model, the pairs of numbers of interest are (P_V, Q_V). Generic notation familiar to some from mathematics classes denotes pairs as (x, y). Most people are familiar with this basic concept from knowledge of map coordinates. A point on a map is described by its *coordinates* : a pair of numbers, one of which specifies latitude and the other longitude. To establish the one-to-one correspondence between points in a plane and pairs of numbers, start with a horizontal line in a plane, extending indefinitely to the left and the right. In generic notation, this line is known as the $x - axis$. For the model of this section, we might want to think of this axis as the $P_V - axis$ (although later in this section, for quirky reasons of historical developments in economic thought, we will "switch" the notation of this axis). A reference point O on this axis and a unit of length (e.g., price of a bottle of wine) are then chosen. The axis is scaled by this unit of length so that the number zero is attached to point O, the number $+a$ is attached to the point a units of length to the right of O, the number $+2a$ is attached to the point $2a$ units to the right of O, the number $-a$ is attached to the point a units to the left of O, and so on. In this way, every point on the $x - axis$ corresponds to a unique real number. The perhaps non-intuitive feature of this real number line is that between any two real numbers, no matter how close to each other, there can always to interspersed another real number. This implies that a point on the line takes up no space. For the purposes of economic models, one can think of the real number line as a convenient approximation to numbers that represent small but discrete units.

Now place another straight line vertical in the plane, i.e., at a right angle to the x-axis, through point O. In generic notation, this is the $y - axis$. For the model of this section, we (initially) would think of this as the $Q_V - axis$. It also extends indefinitely up and down. Choose a unit of length, such as quantity of wine per unit of time, and scale the y-axis with this unit, much as with the x-axis. That is, the number b is attached to the point on the y-axis b units above point O, the number $2b$ is attached to the point on the y-axis $2b$ units above point O, and so on.
Now draw a line parallel to the $y - axis$ through point a on the $x - axis$ and a line parallel to the $x - axis$ through point b on the $y - axis$. These two lines intersect at point R, which corresponds to the pair of numbers (a, b). Clearly, for any two real numbers a and b, there corresponds a unique point R, which we denote as $R(a, b)$. Conversely, we say that the coordinates of R are (a, b). The following figure illustrates the point associated with the pair $(2, 1)$.



The Cartesian Plane

The two axis divide the plane into four quadrants labeled I, II, III, and IV, where standard terminology denotes quadrant I as that section for which all points have two positive coordinates. This quadrant is of most use in economics because most economic variables, such as prices and quantities, are inherently non-negative numbers. Quadrant II has points with signs $(-, +)$, quadrant III has points with signs $(-, -)$ and quadrant IV has points with signs $(+, -)$.

Graphs of functions Because of the one-to-one relationship between pairs of numbers and points in a plane, we can now depict the functional relationship between S_i^d and P_S captured in Table 3.1 (for the stipulated fixed values of the other exogenous variable) and expressed as the set of ordered pairs $\{(\frac{1}{4}, 1), (1, \frac{1}{4}), (2, \frac{1}{8})\}$. The first member of the pair is measured along the horizontal axis and the second member along the vertical axis. These points are

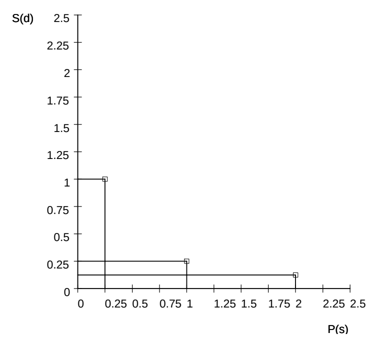


Figure 3: Pairs of (P_S, S_i^d)

depicted in Figure 3.

As noted, expressing this information in table form is cumbersome and limiting. Consequently, we like to express this information in equational form, either in words or in functional notation so as to succinctly capture the relationship between the endogenous and exogenous variables in the demand relationship in a symbolic form. We also like to allow the variables to take any value in some specified segment of the real number line, because this facilitates representation of relationships as continuous curves. This representation provides a much more concise way of expressing all the members of a function which has many members, as do most of the functions in which we are interested.

Consider, for example, the function:

$$S_i^d = \frac{1}{4P_S}.$$

We can immediately check that the three points from our table are members of this function by substituting the three values of P_V into the equation $S_i^d = \frac{1}{4P_S}$ and confirming that the values of S_i^d that result do indeed correspond to the associated points from the table. From this function, we could construct as detailed a table as we would like by picking values of P_S and calculating $\frac{1}{4P_S}$ to get the associated value of S_i^d. If we wanted to construct a table that incorporated every possible pair, though, it would be an infinitely large table, because there are an infinite number of real numbers that lie in the interval between zero and any positive real number. This means that the functional relationship will have an infinite number of set members. The graph of this set of points, though, is easily depicted in the Cartesian plane as a continuous curve. We can think of this graph as if constructed by drawing a line with a pencil without lifting the pencil from the paper. Such a graph is depicted in Figure 4.

As noted earlier, the point here is not that we think most economic variables are measured continuously. The assumption of continuous measurement is simply convenient and **innocuous**. An innocuous assumption is one that can

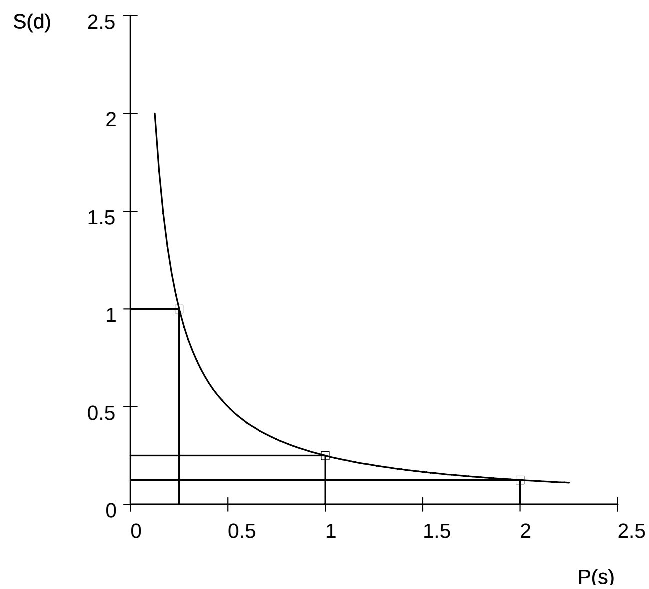


Figure 4: $S_i^d = \frac{1}{4P_S}$

be replaced by a more realistic or complicated assumption without affecting the main conclusions or implications drawn from the model. When we describe an assumption as innocuous, we are appealing to the authority of the economics profession at large, members of which have in fact determined through research that the assumption doesn't affect the main conclusions of the theory.

innocuous assumption: one that can be replaced by a more realistic assumption without changing the main conclusions drawn from the model.

Depicting the logical interrelations: curve-shifting and escaping "Flatland." Our graph of the three representative numbers of the demand function $S_i^d = f(P_S, Y_1)$ depicted pairs of numbers in the $P_S - S_i^d$ Cartesian plane, as did our graph of the specific function $S_i^d = \frac{1}{4P_S}$ from which the three numbers in the table could have been picked. We could only do this by "holding constant" the values of the other exogenous variable of this sub-model, namely Y_i.

That is, our demand function is really a three - variable function: a collection of *ordered triplets* of numbers that tell us the unique value of the variable S_i^d associated with any pair of numbers assigned to the variables P_S and Y_i, respectively.

Depicting three - variable functions on a two-dimensional piece of paper or screen ("flatland") is possible, but difficult for many people without aid of computer-generated drawings. As a consequence, economists have developed techniques for collapsing the information in our three - variable function into a two-dimensional graph. The technique imposes the ceterus paribus assumption on Y_i and then plots in the two-dimensional P_S- S_i^d plane the curve relating

S_i^d to P_S for these given other exogenous variable values.

The placement of this curve in the P_S- S_i^d plane would change, though, if there were a different value for Y_i. To understand how this placement changes, it helps to work through an example in which we assume a **specific functional form** for our demand function.

Assumption of specific functional forms To help make an abstract equation like (3.1) or (3.2) more concrete, we frequently specify the relationship as a particular explicit equation. This is known as picking a *specific functional form to serve as an example.* We will work through two examples, each with a different but specific functional form.

The linear case First consider a **linear function** for individual i:

$$S_i^d = a_{0,i} - a_{1,i}P_S + a_{2,i}Y_i,\ a_{0,i} \geq 0,\ a_{1,i} \geq 0,\ a_{2,i} \geq 0. \tag{3.2}$$

The lower - case $a's$ in (3.2) are examples of **parameters**. We index them by i to emphasize that they are associated with a particular individual. As noted, they are like exogenous variables in that they symbolize numbers that are determined outside of the model, and they are usually assumed to be constant. All of them in this example are restricted to be non-negative so that equation (3.2) conforms to the assumptions we made about how S_i^d moves in response to changes in the values of the exogenous variables, i.e., as P_S gets larger, ceterus paribus, S_i^d gets smaller.

At this point, we need to address one of the peculiarities of the economics profession in terms of how it translates equations into graphs. In keeping with a long tradition in economics, the price of sugar, measured in units of currency/unit of sugar, is measured along the vertical axis, while the quantity/unit of time is measured along the horizontal axis. Note that this means we are measuring the endogenous variable S_i^d along the horizontal axis and the exogenous variable P_S along the vertical axis. This tradition is sometimes confusing to students who have been schooled in high school algebra to graph functions with the dependent (endogenous) variable on the vertical axis and the independent (exogenous) variable on the horizontal axis, as we did in Figures 3 and 4.

For the linear functional form of (3.2), one can use algebra to express the demand relationship with P_S on the left-hand-side of the equality sign and all other variables on the right:

$$P_S = (\frac{a_{0,i}}{a_{1,i}}) - (\frac{1}{a_{1,i}})S_i^d + (\frac{a_{2,i}}{a_{1,i}})Y_i. \tag{3.3}$$

We call this the *inverse form* of the demand curve. In general functional notation, we would write this as follows:

$$P_S = f_i^{-1}(\overset{-}{S_i^d}, \overset{+}{Y}_i) \tag{3.4}$$

where again the algebraic signs above each argument tells us the direction in which the value of left-hand- variables moves when the value of a right-hand-side argument goes up. These signs can be inferred from our fundamental assumptions embodied in equation (3.1). For the linear functional form, this is straightforward: we can infer the signs by looking at the signs of the coefficients on the right-hand-side of (3.3). The symbol f_i^{-1} is the traditional way of describing an inverse function, and is just another symbol that is to be read "...is a function of...". Again, let us emphasize that this inverse form better corresponds to the traditional graphical approach in economics of measuring own price on the vertical axis and quantity demanded per unit of time on the horizontal axis.

We could go one step further in making our examples concrete by specifying actual numbers as examples of parameters. For example, we could specify a linear demand curve as:

$$S_i^d = .5 - P_S + 2Y_i. \tag{3.5}$$

Concept check: What are the specific values assumed for the parameters $a_{0,i}$, $a_{1,i}$, and $a_{2,i}$ in this example?

Occasionally, this further step away from the abstract can provide a useful "mental hook" on which to hang a concept. It also further helps us understand the pedagogical technique of "curve-shifting" that economists frequently use to teach models. Economists will frequently say something along the lines of : "If, ceterus paribus, income increases, this shifts the demand curve out to the right." To understand better what this means, first rearrange equation (3.5) in inverse form:

$$P_S = .5 - S_i^d + 2Y_i. \tag{3.6}$$

Now imagine that hypothetical numerical values for the exogenous variable Y_i is three - fourths ($\frac{3}{4} = .75$). The demand curve represented by equation (3.5) could now be written as

$$\begin{aligned} S_i^d &= .5 - P_S + 2 \times \overbrace{(.75)}^{Y} \qquad (3.7) \\ &= 2 - P_S. \end{aligned}$$

Equivalently, (3.6), would be

$$P_S = 2 - S_i^d. \tag{3.8}$$

With these numerical values for the exogenous variables, the inverse form is a straight line with slope of minus one (-1) and intercept of two (2). This is depicted in Figure 5.

Now imagine a new hypothetical value of income for individual i of one (1), i.e., $Y_i = 1$. The graphical representation of this new curve is again a straight line with slope minus one (-1), but with intercept of two-and-one-half ($2\frac{1}{2}$). This new curve is parallel to the old one, but has "shifted up" or, in equivalent language, "shifted out," so that associated with every value of S_i^d there is now a higher value of P_S. These two curves are actually depicted in Figure 6, where the black line depicts the case where $Y = \frac{3}{4}$ and the red line depicts the case where $Y = 1$.

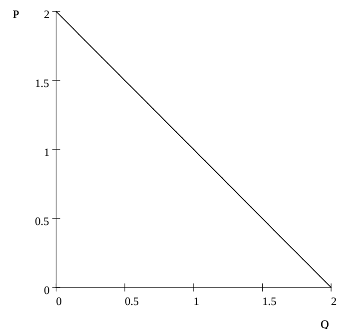


Figure 5: Inverse linear demand function: $Y_i = \frac{3}{4}$.

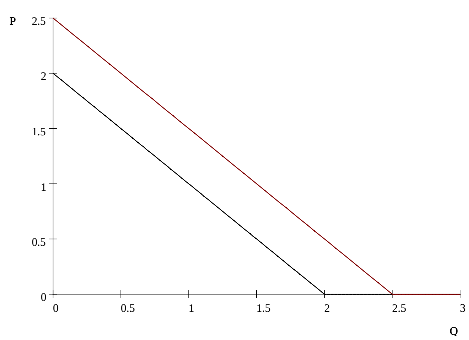


Figure 6: Inverse demand functions with different Y_i's.

The linear function

The linear function used in this example is frequently used in other economic applications. In generic notation, we frequently write a linear function as follows:

$$y = mx + b$$

where y and x are variables and m and b are numbers known as the parameters of the function. When written in this fashion, we say the function is written in slope-intercept form. The y-axis intercept is the value of y when $x = 0$. The slope of the function is symbolized by m , and tells you the change in the y-variable for any given change in the x-variable as you move from one point on the line to another. To elaborate, suppose you know that a point $P_1(x_1, y_1)$ lies on the above line and a point $P_2(x_2, y_2)$ also lies on the line, where the x_i's and y_i's stand for particular numerical values, such as $x_1 = 1,\ x_2 = \frac{1}{2}, y_1 = 1,\ y_2 = 1\frac{1}{2}$.For these particular values, the two points P_1 and P_2 would be $(1,1)$ and $(\frac{1}{2}, 1\frac{1}{2})$, respectively. The *change in* y from point P_2 to point P_1, symbolized as either $y_2 - y_1$ or Δy, is defined as the value of the $y-$coordinate of point P_2 minus the value of the $y-$coordinate of point P_1. The *change in* x, symbolized as either $x_2 - x_1$ or Δx, is defined as the value of the $x-$coordinate of point P_2 minus the value of the $x-$coordinate of point P_1. For the example points $P_1(1,1)$ and $P_2(\frac{1}{2}, 1\frac{1}{2})$, $\Delta y =\ \frac{1}{2}$, and $\Delta x = -\frac{1}{2}$. For any arbitrary value of $y \neq y_1$, and arbitrary value of $x \neq x_1$, the ratio $\frac{y-y_1}{x-x_1}$ is known as the slope, m. For the example of our linear demand curve, the intercept is 2 and the slope is -1. Note that the change in the y-value coordinates of any two points on the line divide by the change in x-value coordinates for these two same points is always the same number, m.

The multiplicative/exponential case To make sure this concept of curve-shifting is clear, now consider another example in which we use a different functional form. Consider the following specification:

$$S_i^d = \alpha_{0,i}(P_s)^{(-\alpha_{1,i})}(Y_i)^{(\alpha_{2,i})}, \alpha_{0,i}, \alpha_{1,i}, \alpha_{2,i} \geq 0. \tag{3.9}$$

where the $\alpha' s$ are parameters. Remembering the rules of exponents from high school algebra (reviewed in the associated box), this can be expressed in inverse form as

$$P_S = (\frac{S_i^d}{\alpha_{0,i}})^{\left(-\frac{1}{\alpha_{1,i}}\right)}(Y_i)^{(\frac{\alpha_{2,i}}{\alpha_{1,i}})}. \tag{3.10}$$

We will see that this example of a demand curve, like the linear example above, satisfies the general assumptions we made about how the quantity demanded per unit of time is affected by the price and income variables. That is, the quantity demanded changes in the stipulated direction for an increase in each of the price and income variables.

Exponents

The expression 6^2 means that six (6) is to be raised to the second power, which means multiply six (6) time six (6). In general, we have

$$x^n \equiv \underbrace{x \times x \times x \times ... \times x}_{n\ terms}$$

Rules of exponents

1. $x^n \times x^m = x^{n+m}$ (example: $x^2 \times x^5 = x^7$)
2. $\frac{x^n}{x^m} = x^{n-m}$
3. $x^{-n} = \frac{1}{x^n}$
4. $x^0 = 1$
5. $x^{\frac{1}{n}} = \sqrt[n]{x}$
6. $(x^n)^m = x^{nm}$
7. $x^n y^n = (xy)^n$

As with the linear example, let us assume the following specific numerical values for the $\alpha' s$ and the exogenous variables other than P_S. For the parameters, assume they take the following values:

$$\alpha_{0,i} = 1,\ \alpha_{1,i} = 1,\ \alpha_{2,i} = 2.$$

For the exogenous variable, assume that $Y_i = \frac{1}{2}$. With these values, equation (3.10) becomes:

$$\begin{aligned} P_S &= \left(\frac{S_i^d}{\underbrace{1}_{\alpha_{0,i}=1}}\right)^{\overbrace{-1}^{\frac{-1}{\alpha_{1,i}}}} \left(\overbrace{\frac{1}{2}}^{Y_i}\right)^{\overbrace{2}^{\frac{\alpha_{2,i}}{\alpha_{1,i}}=2}} \\ &= \frac{1}{4S_i^d} \end{aligned} \tag{3.11}$$

For someone less familiar with a function of this form, it may help to remember that the points that comprised our earlier tabular example of points of a demand function were members of this function. This function is displayed in Figure 7.

Now let us see how this curve shifts in response to a ceterus paribus change in Y_i. As with the linear example, now, instead of having $Y_i = \frac{1}{2}$, let $Y_i = 1$. The inverse demand function then becomes

$$P_S = \frac{1}{S_i^d} \tag{3.12}$$

You may want to plot a few points for this function. Figure 8 superimposes the graph of equation 3.15, depicted in red and placed "higher" along the vertical

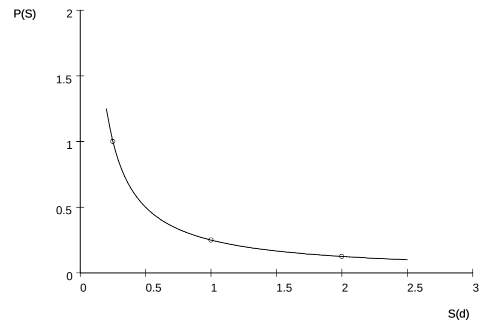


Figure 7: $P_S = \frac{1}{4S_i^d}$

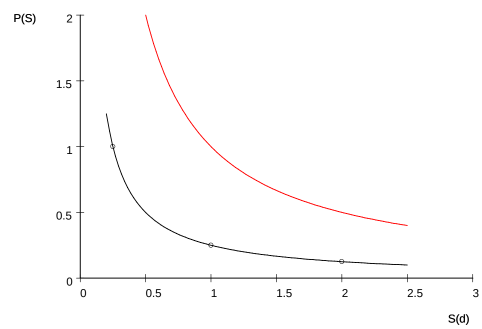


Figure 8: $P_S = \frac{1}{4S_i^d}$; $P_S = \frac{1}{S_i^d}$

axis or "farther out" along the horizontal axis than the graph of 3.14.ii, on the graph of 3.14.ii to illustrate this "shift."

Before moving on, we need to emphasize that the use of specific functional forms is mostly a pedagogical device: it permits us to make concrete the less constraining but harder-to-visualize general model. Specific functional forms are seldom an implication of theory. However, to be useful, they must be consistent with the theory. In the above demand curve examples, consistency of the specific functional forms with the theory is embodied in the sign restrictions on the a's and α's. Even though the different functional form assumptions gave rise to different graphs - one is a straight line, the other has curvature - both satisfy the theoretical restriction that requires the curve to slope down.

Also before leaving this introductory discussion of functional form, we note two other reasons besides the pedagogical one that economic models are expressed with specific functions. First, most general functions of interest to economists can be approximated by one or another specific functional form. Second, when economists use data to estimate an economic model, perhaps for forecasting purposes, they often need to assume a particular functional form. In fact, the "multiplicative case" of a demand curve is often labeled a "constant elasticity" demand curve because this functional form has a particular property - "constant elasticity" - that frequently fits economic data quite well. The hope of the economist is that the specific functional form chosen is in fact a good approximation to whatever the unknown "true" functional form might be.

Curve-shifting: the general case and "bidirectional" logic. As noted, often all that we know about a function are its qualitative properties, as described, for example, by equation (3.2). With only this limited qualitative information, how do we carry out the curve-shifting exercise that we just did with specific functional forms?

First note that by holding constant the values of the exogenous variable Y_i at some particular value denoted by, say, $(Y_i)_0$, we have created a two-variable function that conceptually tells us an associated value of S_i^d for every permissible value of P_S. The use of a numerical subscript, in this case zero or "naught," is standard notation for indicating that we are contemplating a particular value of a variable. We say "conceptually" to emphasis that we have no specific function in mind. We do know, though, that our behavioral assumption (1) implies that this function must associate lower values of S_i^d with higher values of P_S . This means the graph of this function, with S_i^d plotted on the vertical axis and P_S plotted on the horizontal axis, is a curve with negative slope. It also implies that the graph of the inverse demand function, with P_S measured on the vertical axis and S_i^d measured on the horizontal axis, will also be a curve with negative slope.

In the tradition of economics, let us focus on the graph of the inverse demand curve, and consider the inverse demand curve when the value of the exogenous variable Y_i is $(Y_i)_0$. Contemplate now a particular value of S_i^d along this curve, denoted by $(S_i^d)_0$, and the associated value of P_S, denoted by $(P_S)_0$. Ask the hypothetical question: What would happen to S_i^d if, at $P_S = (P_S)_0$, Y_i were to increase, i.e., Y_i were to take a value greater than $(Y_i)_0$? The answer to this hypothetical question is given by our behavioral assumption that, ceterus paribus, an increase in income increases the quantity demanded. At the higher value of Y_i, it must be that $S_i^d > (S_i^d)_0$.

Now, there is nothing special about the value $(P_S)_0$. We could have carried out the same thought experiment for any of the permissible values of P_S, and for each value the same logic would apply: at the higher value of Y_i, it must be that $S_i^d > (S_i^d)_0$. Hence, at every value of P_S, the graph of the inverse demand function would have "shifted up." in response to an increase in Y_i.

We could also imagine going through the same exercise with the following change: instead of asking about what happens to S_i^d at particular values of P_S when Y_i increases, ask what must happen to P_S at particular values of S_i^d when, ceterus paribus, Y_i increases This question is perhaps less intuitive because our behavioral assumption is that people *choose* quantity per unit of time based on the values taken by exogenous variables. This new question almost seems to ask: what price does individual i choose at a given value of S_i^d when there is an increase in Y_i, ceterus paribus? But this is not correct. What we are really asking is: if we observe a person consuming the same quantity in the face of an increase in Y_i, then what must have happened to P_S? Because an individual would consume more with higher Y_i and unchanged P_S, but we observe (or "hold constant" in our thought experiment) an unchanged S_i^d, then it must be that P_S has gone up so as to offset the increased demand from the increase in Y_i. Hence, for every value of S_i^d, the graph will have "shifted out."

To recap, with only information about the qualitative responses of individuals' demands to changes in exogenous variables, we can still determine via thought experiments in which direction the demand curve will shift. We call this **bidirectional logic** because our strategy was to imagine holding constant either the variable measured on the vertical axis and asking what must happen to the variable measured on the horizontal axis, or holding constant the variable on the horizontal axis and asking what must happen to the variable on the vertical axis. In the former case, the curve "shifted out" while in the latter the curve "shifted up." Both depict the same phenomena.

5.2.3 The "market" or "aggregate" demand curve: summing up over individuals

Our sub - model of demand is almost complete. We have developed a model of an *individual's* demand curve, but not a model of the total demand for sugar. In the sugar market, what matters to suppliers is the total (or "market" or "aggregate") demand function. To get this market demand function, we simply add up the individual demand functions. If we denote market demand by the symbol S^d, then we have that S^d is the sum of all the individual demand curves:

$$\begin{aligned} S^d &= S_1^d + S_2^d + ... + S_i + ... + S_I \\ &= f_1(P_S; Y_1) + f_2(P_S; Y_2) + ... + f_I(P_S; Y_I) \\ &\equiv f(P_S; Y_1, Y_2, ...Y_I). \end{aligned} \tag{3.13}$$

The subscripts denote "individual one, individual two," and so forth, and I symbolizes the total number of individuals who have demands for sugar. For example, individual one might be someone named "Alex," individual two might be someone named "Bobby," and so forth, and I might be 3000. For notational simplicity, we have defined a new function, $f(P_S; Y_1, Y_2, ...Y_I)$, to denote the aggregate or market demand function.

Graphically, we construct market demand functions by "adding up" along the horizontal axis the individual inverse demand functions. For example, suppose there were only two demanders: Alex and Bobby[2]. Assume their demand curves are specified as:

Concept check: By writing their demand functions in this fashion, about what variable are we making the ceterus paribus assumption?

[2] The asumption of only two individuals is obviously unrealistic, but is useful because we can mentally "keep track" of two individuals.

$$\begin{aligned} S_A^d &= \frac{1}{P_S}; \\ S_B^d &= \frac{2}{P_S}. \end{aligned}$$

Their inverse demand functions would be:

$$P_S = \frac{1}{S_A^d} \qquad \text{(Alex's inverse demand)}$$

$$P_s = \frac{2}{S_B^d} \qquad \text{(Bobby's inverse demand)}$$

Market demand would thus be:

$$\begin{aligned} S^d &= S_A^d + S_B^d \\ &= \frac{1}{P_S} + \frac{2}{P_S} \\ &= \frac{3}{P_S}. \end{aligned}$$

Notice that the market demand curve shares the same qualitative features as do the individual demand curves: as price increases, quantity demanded falls.

The inverse market demand curve is

$$P_s = \frac{3}{S^d}.$$

The depiction of the inverse market demand function as equal to the "horizontal addition" of the individual demand curves is depicted in Figure 9. In this figure, the curve closest to the origin is Alex's inverse demand curve, the curve next to Alex's inverse demand curve is Bobby's, and the thicker curve farthest out from the origin is the sum of these two individual inverse demand curves.

5.3 Supply

Now consider the sub-model for supply. What are the important variables that influence the decisions of firms about how much of a good to produce over a given time period? Twin underlying assumptions, one about behavior and the other about technology, motivate the answer to this question. One assumption describes the mainspring of firm motivation: firms attempt to maximize profits. A second assumption is that there exists a well-defined technology called the production function that tells how much output per unit of time can be produced for any given quantity of inputs, e.g., labor, per unit of time. Given these assumptions, a full-blown detailed model of firm decision-making tells us that the quantity produced per unit of time is an increasing function of the per - unit revenue the firm receives for its output, and a decreasing function of the price of inputs into the production process. Again assume we are concerned with production of sugar. Denote the quantity of sugar produced by a particular

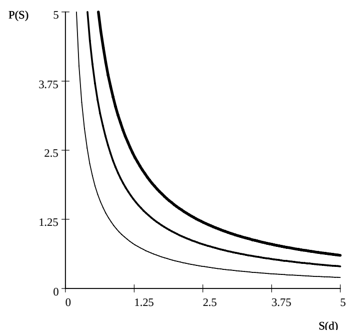


Figure 9: Market and individual inverse demands

firm as S_j^s, where the subscript j denotes that we are describing supply by the "j^{th}" firm. Assume the only inputs into the production process are labor and land, with associated prices w (for wage) and r (for rent). For the firm, the per - unit revenue it receives is the sum of the market price, P_S, and any per - unit subsidy it receives, which we will denote by the Greek letter σ_j ("sigma sub j"). The quantity produced is endogenous, and the price of sugar, the amount of any subsidy received, and the price of inputs are exogenous. The shorthand expression for the relationship between these variables is the individual firm supply schedule

$$S_j^s = g_j(P_S \overset{+}{+} \sigma_j, \overset{-}{w}, \overset{-}{r}). \tag{3.14}$$

Analogous to the demand schedule, $P_S + \sigma_j$, w, and r are the arguments of the firm supply function.

For our purposes, we can ignore the effects of input prices on the supply function and focus on the effects of changes in the per - unit subsidy σ_j on the inverse supply function. That is, we will ignore w and r for the rest of our analysis. Another way of thinking about what this simplification entails is to think of us as simply imposing the ceterus paribus assumption on w and r.

Our symbolic representation in general functional notation of an individual firm's supply function is, with this simplification, thus:

$$S_j^S = g_j(P_S \overset{+}{+} \sigma_j) \tag{3.15}$$

We could also write this in inverse form:

$$P_S + \sigma_j = g_j^{-1}(\overset{+}{S_j^S}). \tag{3.16}$$

Remember, expression of either a demand or supply functional relationship in inverse form conveys no new information: It simply arranges variables in a

This last expression for the j^{th} firms inverse supply function turns out to be especially useful for us, because we can re - write it with just P_S on the left - hand - side:

$$P_S = g_j^{-1}(\overset{+}{S_j^S}) - \sigma_j \tag{3.17}$$

Writing the function this way partially obscures the knowledge we have of the relationship between P_S, σ_j and the per - unit revenue received by the firm, $P_S + \sigma_j$. But it does prepare us for a graphical depiction of supply curves on a diagram with the same variables on the axes as in the diagram of the market demand curve. This is important because we eventually want to superimpose on one diagram both the market supply and the market demand functions. Such a diagram proves useful for understanding and depicting the *solution* of the model.

Again, use of a specific functional form might make clear the relationship between the supply function written with the lone argument $P_S + \sigma_j$ and the supply function written with P_S and σ_j as separate arguments.

Consider the linear parametric specification of firm $j's$ supply function:

$$S_j^S = b_{0,j} + b_{1,j}(P_S + \sigma_j),\ b_{1,j} > 0. \tag{3.18}$$

This, of course, can be re - written as

$$S_j^S = b_{0,j} + b_{1,j}P_S + b_{1,j}\sigma_j. \tag{3.19}$$

In inverse form, this equation would be:

$$P_s = -\frac{b_{0,j}}{b_{1,j}} + \frac{S_j^S}{b_{1,j}} + \frac{\sigma_j}{b_{1,j}}. \tag{3.20}$$

To be more concrete, we could assume the parameters of this equation have the values:

$$b_{0,j} = 0,\ b_{1,j} = 1$$

so that firm j's inverse supply function would be

$$P_s = S_j^S + \sigma_j.$$

For any given value of σ_j, the graphical depiction of firm j's inverse supply function would be an upward-sloping curve with P_S on the vertical axis and quantity of sugar per unit of time on the horizontal axis. The curve "shifts up" (or, in equivalent language, "shifts back") as the subsidy σ_j takes on larger and larger values. Examples are depicted in Figure 10 , with the red line depicting the inverse supply function associated with the lower value of σ_j.

Market supply functions Much as with the sub - model of demand, we need to combine individual firm supply functions to create a market supply function. And much as with the demand sub - model, we create a market supply function by adding up individual supply functions. That is, if we symbolize

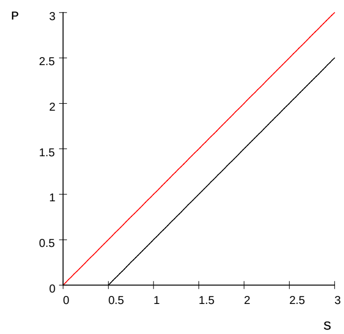


Figure 10: Individual inverse supply functions

market supply by S^s, the market supply function is just the sum of all the individual firm supply functions:

$$S^s = \sum_{j=1}^{j=J} S_j^s$$

where we use the *summation symbol* "$\sum$" to indicate the addition of all the individual firms' supply functions, of which there are J. That is,

$$\begin{aligned} S^s &\equiv \sum_{j=1}^{j=J} S_j^s = S_1^s + S_2^s + ... + S_J^s \\ &= g_1(P_s, \sigma_1) + ... + g_J(P_S, \sigma_J). \end{aligned} \tag{3.21}$$

Because the market supply function is defined as the sum of the individual firms' supply functions, for notational ease we can denote it as a new function, g:

$$S^s = g(P_S; \sigma_1, ..., \sigma_J) \tag{3.22}$$

For our purposes, suppose there are only two firms: one in the European Union that receives a subsidy σ_{EU}, and one in "the rest of the world" (denoted ROW in international economics jargon) that does not receive a subsidy. Further suppose the EU firm has the following supply function:

$$S_{EU}^s = P_S + \sigma_{EU}.$$

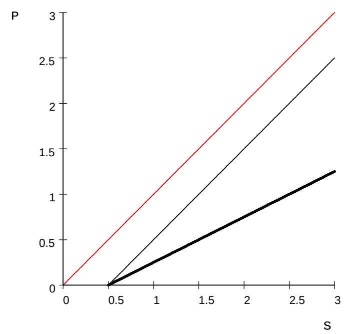


Figure 11: Market and individual inverse supply

In inverse form this would be

$$P_S = S^s_{EU} - \sigma_{EU}.$$

Assume the ROW supply function is:

$$S^s_{ROW} = P_S.$$

Adding up, we have that the world, or market, supply function of sugar is

$$S^s = 2P_s + \sigma_{EU}.$$

In inverse form this would be

$$P_s = \frac{1}{2}S^s - \frac{1}{2}\sigma_{EU}$$

The inverse supply functions are depicted in Figure 11 for $\sigma_{EU} = \frac{1}{2}$. The red line depicts the ROW inverse supply function, the thin black line depicts the EU inverse supply function, and the thick black line depicts the market inverse supply function.

This completes our discussion of the sub-models of demand and supply. Note how each sub-model answers the question: What happens to the value of the endogenous variables in the demand sub-model, quantity demanded per unit of time, quantity supplied per unit of time in the supply sub-model - when the values of the exogenous variables change? We now combine these sub-models into an equilibrium model of demand and supply. While in each of the above sub-models the price of sugar was taken as exogenous by the decision-makers, in the equilibrium model this variable will be taken as endogenous.

5.4 Equilibrium

The concept of "equilibrium" is that we have found a "rest point" where, in the absence of any changes in either the values of exogenous variables or in the specification of the logical interactions among these variables, the values of the endogenous variables are unchanging. For our partial-equilibrium model, this "rest point" occurs when the quantity demanded equals the quantity supplied. In symbolic notation, this *equilibrium condition* is:

$$S^d = S^s \tag{3.23}$$

With the addition of this equilibrium condition, the specification of our model is complete. When we have a simple listing of all the equational statements of a model, where each equation represents either a behavioral relationship, an identity, definition, or technical relationship, or an equilibrium condition, then we say the model is written in *structural form*. The structural form of our simple demand-supply model is simply the collection of the behavioral equations representing the demand function and the supply function, and the equilibrium condition that quantity demanded equals quantity supplied. For emphasis, we collect the structural equations of our model here:

$$\begin{aligned}
S_i^d &= \sum_{i=1}^{i=I} f_i(\overset{-}{P}_S, \overset{+}{Y}_i) \equiv f(\overset{-}{P}_S, \overset{+}{Y}_1, \overset{+}{Y}_2, ..., \overset{+}{Y}_I) && (3.13) \\
S_j^s &= \sum_{j=1}^{j=J} g_j(\overset{+}{P}_S, \sigma_j) \equiv g(\overset{+}{P}_S, \overset{+}{\sigma}_1, \overset{+}{\sigma}_2, ..., \overset{+}{\sigma}_J) && (3.22) \\
S^d &= S^s && (3.23)
\end{aligned}$$

Each equation spells out an economic assumption of the model, telling us something about postulated behavior of individual economic units - ((3.13) and (3.22)) - or markets - (3.23). That is, the structural model lays bare the economic framework of the model.

Our goal, though, is to use the model to answer the canonical questions we posed earlier: what is the relationship between the values of the exogenous variables and the values of the endogenous variables? To this end, we now develop various ways that we can answer this question via a solution of the model.

6 Solving the model

Generally, there are two presentations of the solution of this model: graphical and mathematical. For pedagogical purposes, we will use graphical methods whenever possible ("a picture is worth a thousand words"). We also provide the in-depth mathematical solution in this case, though, so one can see most clearly the connection between equations and graphs.

6.0.1 Solving the model without graphs

A strategy or "protocol" often used to solve a model is to first substitute behavioral relationships into the equilibrium condition. This allows us to solve for the market - clearing, i.e., equilibrium, market price and quantity. Having solved for the market price, the individual equilibrium quantities demanded and supplied can then be determined.

For expositional simplicity, we will from here on assume there are just two firms in the model, the firm in the European Union and the firm in the "rest of the world." Thus, the only exogenous supply variable is σ, the subsidy paid to the EU firm. Substitution of equation (3.13), the demand schedule, and equation (3.22), the supply schedule, into equilibrium condition (3.23) tells us implicitly what price is necessary to equate quantity demanded to quantity supplied:

$$f(P_S; Y_1, Y_2, ...Y_I) = g(P_S; \sigma) \tag{3.24}$$

We separate the arguments in each function by a semi-colon to denote that the variables to the right of the semi-colon in each function are exogenous. This emphasizes that (3.24) is a single equation in which the only endogenous variable is P_S.

Implicit and explicit functions

We say that this substitution tells us *implicitly* the equilibrium price that clears the market because (3.24) is an example of an implicit function. Up until now, we have encountered mostly functions of the form $y = f(x)$. In equations like this, the variable y simply appears by itself on the left hand side of the equation, and expressions which only involve the variable x appear on the right hand side. That is, there is no intermingling of y and x on either side of the equality sign, and y appears as the variable itself and not as some function of the variable such as, for example, y^2 , or perhaps $y - 6$. We say that such an equation tells us *explicitly* what the value is of y for any permissible value of x. For example, the following equation expresses P_S *explicitly* as a function of S_i^d:

$$P_S = \frac{1}{4S_i^d}.$$

An implicit function, on the other hand, may have variables intertwined and showing up on either side of the equality signs. They are equations like

$$\begin{aligned} x + y &= 1; \\ y^2 &= x; \\ x^2 + xy + y^2 &= 3 \end{aligned}$$

As another example, we could multiply both sides of the equation

$$P_S = \frac{1}{4S_i^d}.$$

by S_i^d so as to turn it into an implicit function:

$$P_s S_i^d = \frac{1}{4}$$

Of course, this also means that we can turn this *implicit* equation into an *explicit* equation by multiplying both sides by $\frac{1}{S_i^d}$. For many implicit functions, this transformation from implicit to explicit function can be done. For some implicit functions, this can't be done. But even for these implicit functions (written as $f(x, y) = 0$), it is often the case that we know that there exists a function $y = h(x)$ associated with $f(x, y)$, even though there doesn't exist a *closed form* solution. In either case, of most interest to us is that the functions we will encounter in most economic problems have either closed form explicit solutions or explicit solutions of unknown form.

For the problem at hand, the implicit equation (3.24) can be transformed into an explicit equation with P_S on the left-hand-side of the equality sign and all other variables on the right. When we do this transformation, we often describe this by saying that the equilibrium price of sugar can be “solved out” of equation (3.24) and expressed as a function of just the exogenous variables:

$$\widehat{P}_S = F(\sigma, Y_1, Y_2, ...Y_I). \tag{3.25}$$

This expression for $\widehat{P}_S$ can be substituted into either the demand or the supply function to yield an equation that expresses the equilibrium quantity of sugar produced or consumed per unit of time as a function of just the exogenous variables. For example, if we substituted (3.24) into the supply function (3.22) in place of P_S , we would have

$$\widehat{S^s} = g(F(\sigma, Y_1, Y_2, ...Y_I), \sigma)$$

which we could write as:

$$\widehat{S^s} = G(\sigma, Y_1, Y_2, ...Y_I). \tag{3.26}$$

Finally, because of the equilibrium condition,

$$\widehat{S^s} = \widehat{S^d} \equiv \widehat{S} = G(\sigma, Y_1, Y_2, ...Y_I). \tag{3.27}$$

where we use the “defined as” binary relation symbol (” $\equiv$ ”)to emphasize that the equilibrium quantity per unit of time, $\widehat{S}$, is not a different value for the demand and supply functions. Note that we denote equilibrium values of P_S and S by putting a “hat” over the symbol for the variable. This helps keep clear that when we write P_S and S in the demand and supply functions (without a ”hat”), they are not variables with uniquely determined values, but rather *possible* values of variables. The equilibrium values of these variables, denoted by the ”hat” over them, though, are the particular values that simultaneously satisfy all the equations of the model.

"Solving out"

“Solving out” may not yet be clear to someone without some mathematical sophistication. Again, use of specific functional forms helps make this clearer. For purposes of exposition, consider the examples of linear functional forms for demand and supply, and assume all individual parameters are identical, e.g., $a_{0,i} = a_0/I$ for every individual consumer and $b_{0,j} = b_0/J$ for every firm. Substitution of these equations into (3.24) and use of the tools of ordinary high school algebra yield the explicit function for the equilibrium price:

$$\widehat{P}_S = \frac{a_0 - b_0}{(a_1 + b_1)} - \frac{b_1}{(a_1 + b_1)} \sum_j \sigma_j + \frac{a_2}{(a_1 + b_1)} \sum_i Y_i$$

Now the equilibrium quantity bought and sold can be solved explicitly by substituting the above equation into either the demand or supply function, and applying again the tools of ordinary high school algebra:

$$\widehat{S} = \frac{a_0 b_1 + a_1 b_0}{(a_1 + b_1)} + \frac{a_1 b_1}{(a_1 + b_1)} \sum_j \sigma_j + \frac{a_2 b_1}{(a_1 + b_1)} \sum_i Y_i.$$

where again the ”hat” identifies an equilibrium value. The two above equations are ideally suited to answering the canonical question that we ask of models: what are the values of the endogenous variables for given values of the exogenous variables. If we are given values for the parameters - the a’s and b’s - and are given values for each of the exogenous variables - the Y_i’s and $\sigma's$ - then we can immediately compute the equilibrium values of P_S and S.

Even with knowledge of a specific functional form such as in the above boxed material, we may not know the magnitudes of the various parameters. That is, all we might know about the a’s and b’s is that they are non-negative numbers. Clearly, we cannot ascertain exact numerical values of the solution values of the endogenous variables. In this case the form of the canonical question that we would ask would be: In what directions - larger or smaller - do the values of the endogenous variables move when there is a qualitative change,i.e., an increase or decrease of unspecified magnitude, in the value of an exogenous variable. Because all the parameters were specified as non-negative numbers, we can immediately “sign” these changes by inspection. For example, if $\sum_j \sigma_j$ were to increase, ceterus paribus, then the value of $\widehat{P}_S$ would decrease because the coefficient on $\sum_j \sigma_j$ in the reduced form price equation, namely $\frac{-b_1}{a_1+b_1}$, is negative. In contrast, the value of $\widehat{S}$ would increase because the coefficient on $\sum_j \sigma_j$ in the reduced - form equation for quantity supplied, namely $\frac{a_1 b_1}{a_1+b_1}$, is positive.

Can we answer these canonical questions even if we don’t have specific functional forms for our demand and supply functions? For this model, we can. What will allow us to do this is the *behavioral* assumptions about what exogenous variables affect demand and what ones affect supply. We emphasize "behavioral" in the preceding sentence to make clear these are assumptions that are manifested in structural equations of the model and not in equations such as (3.25) and (3.26). Equation (3.25) and (3.26) or any of their counterparts that arise from use of a specific functional form of specific example are known as *reduced form* equations. Before seeing how we can answer the canonical questions, let us digress briefly to spell out the relationship between structural and reduced form.

To move from the structural form to the reduced form, we repeatedly substituted structural equational statements about one or another endogenous variables for that endogenous variable in another structural equation. In the above model, we substituted the right-hand side of the structural equation that had S^d on the left-hand-side (equation 3.2) and we substituted the right-hand side of the structural equation that had S^s on the left-hand side (equation 3.19), into the structural equation that imposed the equilibrium condition (equation 3.23). We then substituted the implicit function for P_S that arose out of this first step into the left-hand side of the structural equation for supply. These mathematical manipulations transformed the structural model into the reduced from.

For any model, when this rearrangement is completed, there is one reduced-form equation for each endogenous variable; such an equation has one and only one endogenous variable on the left-hand-side of the equation, with any number of exogenous variables on the right-hand-side. Remember, though, the right-hand-side contains no endogenous variables.

To get from a structural model to the reduced form, the structural model needs to have as many equations as there are endogenous variables. This equality of numbers of structural equations and endogenous variables is in fact a necessary condition for any structural model to be capable of answering the key question we ask of models: what happens to the values of endogenous variables when the value of an exogenous variable changes? This means that counting equations and endogenous variables provides a diagnostic check of whether or not a structural model is “well-specified”, that is, whether or not it can answer our canonical question. As an example, in our demand-supply model, there are three structural equations, namely the demand function, the supply function, and the equilibrium condition, and three endogenous variables: the quantity demanded (S^d), the quantity supplied (S^s), and the price of sugar (P_S). Note again that there is no logical restriction on the number of exogenous variables.

Now, let us return to how we go about answering the canonical question. The first step of the transformation of the structural model into the reduced form was done by equating the demand and supply functions to each other.

This was equation 3.24, reproduced here for convenience:

$$\overbrace{f(\overset{-}{P_S};\overset{+}{Y_1},\overset{+}{Y_2},...\overset{+}{Y_I})}^{S^d} = \overbrace{g(\overset{+}{P_S};\overset{+}{\sigma})}^{S^s} \tag{3.24}$$

The key feature of this equation is that the left-hand side has the single endogenous variable P_S and the exogenous variables $Y_1, Y_2, \ ..., Y_I$, while the right-hand side has the same single endogenous variable P_S but the *different* exogenous variable σ. This reflects our behavioral assumptions about what exogenous variables affect the quantity demanded and quantity supplied, respectively.

Carry out the thought experiment of contemplating a ceterus paribus *decrease* in the value of one of the exogenous variables, say σ, from an initial position of equilibrium. What happens to P_S? There are only three possibilities: It must either stay the same, decrease, or increase. We will trace out the implications of each of these possibilities and find that only one is possible. Because these three possibilities exhaust all cases, the one that is possible is also the correct answer.

We contemplate a *decrease* because it corresponds to the actual proposed policy change by the EU.

Suppose P_S remained the same. This would mean the left - hand side of (3.24), that is, S^d, remains the same number. This of course requires that the right - hand side - S^s- remains that same number. But if σ decreases, ceterus paribus, and P_S remains unchanged, then S^s must have gone down. But if S^s decreased in value, it cannot still be equal to the unchanged value of S^d. We have a contradiction. This eliminates the possibility that P_S remains unchanged in the face of a decrease in σ.

Now suppose P_S decreased. If this happened, the left - hand - side of (3.24) requires that S^d increased. (This is just the behavioral assumption that, ceterus paribus, quantity demanded increases with a decrease in its own price.) If S^d increased, then, in equilibrium, S^s must also have increased. But *behaviorally*, a decrease in P_S, ceterus paribus, decreases the value of the variable S^s. By assumption, the only other variable to change value is σ, and behaviorally a decrease in σ also decreases the value of the variable S^s. Hence, if both P_S and σ decrease, ceterus paribus, then S^s must have decreased. But this means that it cannot be equal to S^d, which must have increased if P_S decreased. We have a contradiction. This eliminates the second possibility that a decrease in σ could lead to a decrease in P_S.

Hence the only possibility left is that P_S increased, so this must be the correct answer. (Remember, one of the three possibilities had to occur.) A diagnostic check on our logic can be done by seeing what happened to S^s from such a rise in price. Our behavioral assumptions about the effects of P_S and σ on S^s tells us that S^s must have risen in value. This means that in equilibrium S^d must also have increased in value. This is consistent with our behavioral assumption about the effect of an increase in P_S on S^d.

Note that if we had simply started our analysis with the possibility that there was an increase in P_S , all we would have proved was that this possibility could not be ruled out. But because we ruled out all other possibilities, this last possibility must in fact be what had to have happened.

Knowing that $\widehat{P}_S$ must have increased in response to a ceterus paribus decrease in σ, we can now figure out what is the response of $\widehat{S}$. Because the exogenous variables in the demand function,i.e., the left - hand side of (3.24), are, by assumption, unchanged, the behavioral assumption that S^d decreases if P_S increases insures that the equilibrium value $\widehat{S}$ (which must equal both the quantity demanded, the value of the left-hand side of (3.24), and the quantity supplied, the value of the right-hand side) has decreased.

Finally, with knowledge of how $\widehat{P}_S$ has changed in response to the hypothesized change in σ, we can determine the change in each individual's quantity demanded and the change in each firm's quantity supplied. This will complete our description of the changes in the values of all the endogenous variables in the model engendered by a hypothetical change in an exogenous variable.

For the quantity demanded by each individual, this last step is straightforward. The quantity demanded by each individual in equilibrium is given by their individual demand functions, $f_i(\widehat{P}_S; Y_i),\ i = 1, 2, ...I$. Knowing $\widehat{P}_S$, and knowing Y_i (because it is exogenous), we obviously know $\widehat{S}^d_i = f_i(\overset{-}{\widehat{P}_S}; Y_i)$. Hence, if we contemplate a ceterus paribus decrease in σ, the European Union sugar production subsidy, we know that $\widehat{P}_S$ would increase, and thus $\widehat{S}^d_i$ would decrease. This would be true for every individual's quantity demanded per unit of time.

On the supply side, the results are a little less straightforward to obtain. The equilibrium quantity supplied by the Rest of the World firm is described by its individual supply function:

$$\widehat{S}^s_{ROW} = g_{ROW}(\overset{+}{\widehat{P}_S}).$$

Following a hypothetical decrease in σ, we know that $\widehat{P}_S$ increases. Hence, sugar production by the Rest of the World firm would increase.

But the quantity supplied per unit of time by the European Union firm is given by its supply function:

$$\widehat{S}^s_{EU} = g_{EU}(\overset{+}{\widehat{P}_S}; \overset{-}{\sigma}).$$

The tricky part here is that the contemplated decrease in σ is assumed, *ceterus paribus*, to decrease European Union sugar production. But the *equilibrium* price, $\widehat{P}_S$, would have increased, which, ceterus paribus, would lead to an increase in production.

Which "ceterus paribus" effect is stronger - the increase in production from the increase in price or the decrease in production due to the decrease in subsidy - when ceterus paribus does not apply to the behavioral relationship in question? To answer this, we must make use of our knowledge of what would have happened to total world production and to the ROW production. In equilibrium, we have already shown that world production of sugar per unit of time would have decreased in response to a reduction in the EU sugar subsidy. And we have also shown that ROW sugar production per unit of time would have increased. If world production has *decreased* and ROW production has *increased*, it must be that EU production has *decreased*.

It cannot be otherwise: if EU production had stayed the same or increased, then the sum of EU and ROW production would have increased, and hence world production would have increased. This, of course, is a contradiction of what we know: world production would have gone down.

To sum up, we have been able to infer the answer to the "directional" form of our canonical questions about the effects of a decrease in σ on the values of the endogenous variables $\widehat{S}$, $\widehat{P}$,$\widehat{S}_i^d$, $\widehat{S}_{ROW}^s$, and $\widehat{S}_{EU}^s$, even without knowledge of specific functional forms. We might usefully summarize these results in the following reduced - form description of the model:

$$\widehat{P} = F(\overset{-}{\sigma}, \overset{+}{Y_1}, \overset{+}{Y_2}, ..., \overset{+}{Y_I})$$

$$\widehat{S} = G(\overset{-}{\sigma}, \overset{+}{Y_1}, \overset{+}{Y_2}, ..., \overset{+}{Y_I})$$

$$\begin{aligned}
\widehat{S}_1^d &= F_1(\overset{-}{\sigma}, \overset{+}{Y_1}, \overset{-}{Y_2}, ..., \overset{-}{Y_I}); \\
\widehat{S}_2^d &= F_2(\overset{-}{\sigma}, \overset{-}{Y_1}, \overset{+}{Y_2}, ..., \overset{-}{Y_I}); \\
& \quad ... \\
\widehat{S}_I^d &= F_I(\overset{-}{\sigma}, \overset{-}{Y_1}, \overset{-}{Y_2}, ..., \overset{+}{Y_I});
\end{aligned}$$

$$\widehat{S}_2^d = F_1(\overset{-}{\sigma}, \overset{+}{Y_1}, \overset{-}{Y_2}, ..., \overset{-}{Y_I}$$

$$\widehat{S}_{ROW}^s = G_{ROW}(\overset{+}{\sigma}, \overset{+}{Y_1}, \overset{+}{Y_2}, ..., \overset{+}{Y_I})$$

$$\widehat{S}_{EU}^s = G_{EU}(\overset{-}{\sigma}, \overset{+}{Y_1}, \overset{+}{Y_2}, ..., \overset{+}{Y_I}).$$

Note that the directional sign over the arguments $Y_1,\ Y_2,\ ..., Y_I$ in the individual demand functions have a plus sign for an individual's own income but a negative sign for other individuals' incomes. We did not work out these implications, but could with the same logic used for understanding the effects of a change in σ on the various endogenous variables. An ability to understand these signs provides a good check on one's understanding of the solution of this model.

For many people, though, a graphical approach to answering these questions is easier to understand.

6.0.2 Graphical solution

Graphically, market equilibrium is depicted by superimposing on one graph both the demand and the supply curve. Remember, a demand function is a set of points that constitutes a downward-sloping line-not necessarily straight-in the $P_S - S^d$ plane, and a supply function is a set of points that constitutes an upward-sloping line-again not necessarily straight-in the $P_S - S^s$ plane. In terms of inverse demand and supply functions, these curves have the same slopes but are in the $S - P_S$ plane. That is, the vertical axis measures price and the horizontal axis measures quantity. The intersection of these curves depicts the

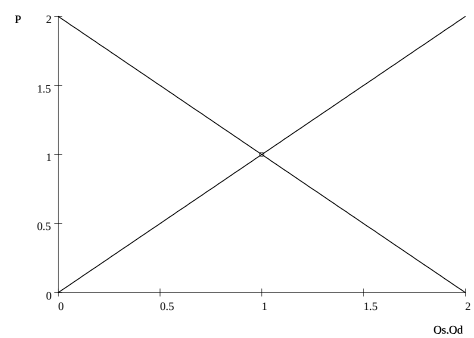


Figure 12: Depicting the demand - supply model

equilibrium price and the equilibrium quantities bought and sold per unit of time.. Notice that the equilibrium is an ordered pair, represented by a point in the plane. This is depicted in Figure 12 as the point $(1, 1)$. The labels on the horizontal axis are both S^d and S^s - one for the demand function and one for the supply function. The point on the horizontal axis where $S^d = S^s$ is that value of S where the quantity demanded equals the quantity supplied. Associated with this equilibrium value $\widehat{S}$ is the equilibrium value $\widehat{P}_S$.

Although the graph uses linear functions to depict this equilibrium, all the information we need to know about the demand and supply curves in order to depict an equilibrium is that they intersect at some point in the positive quadrant of the $S - P_S$ plane. We don't need to know a specific functional form or specific parameter values. Furthermore, without knowing a specific functional form or specific parameter values, we can further characterize the relationship between exogenous and endogenous variables based solely on our qualitative behavioral knowledge captured be the assumed algebraic signs over the arguments in the behavioral equations that constitute the structural model. This is done by applying the "curve-shifting" techniques introduced in our analysis of demand and supply.

The key feature of our model that lets us easily use the diagrammatic analysis to characterize the effects of changes in values of exogenous variables on the values of endogenous variables is the lack of overlap of exogenous variables in the demand and supply behavioral relationships. Because of this, any of our ceterus paribus thought experiments in which we assume a change in the value of just a single exogenous variable results in a shift of only one curve: either the demand (or inverse demand) curve or the supply (or inverse supply) curve.

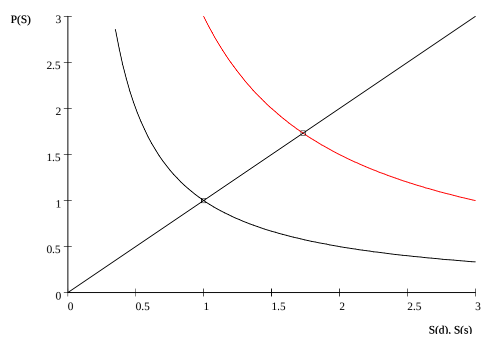


Figure 13: Equilibrium with different income

Hence, a change in, say, income, shifts "out" or "up" the inverse demand curve. The inverse supply curve, though, remains at its initial placement in the $S - P_S$ plane. Consequently, the new equilibrium point is a new pair $(\widehat{S}, \widehat{P}_S)$ that we can think of as representing a movement along the inverse supply curve resulting from a shift in the inverse demand curve. Figure 13 depicts such a shift of the inverse demand curve and the associated movement of the equilibrium pair along the inverse supply curve. In the figure, the new inverse demand curve is depicted as a red line. Clearly, the new equilibrium pair has a higher value for both $\widehat{S}$ and $\widehat{P}_S$.

Other ceterus paribus thought experiments follow the same logic. For example, if σ were to decrease, this would shift up the inverse supply curve while leaving the inverse demand curve unchanged. The new equilibrium could be described as a new pair $(\widehat{S}, \widehat{P}_S)$ that represents a movement along the inverse demand curve resulting from a shift in the supply curve. This new pair would have a higher value of P_S and a lower value of S.

The value of having specified graphical relationships in which only one curve shifts in response to a change in an exogenous variable is that once the direction of the shift is known, the change in the equilibrium values of the endogenous variables is obvious. For our simple partial-equilibrium model of demand and supply, the specification of our behavioral relationships led us naturally to such a diagram. In more complicated models, we frequently manipulate the structural model solely to be able to depict the model in terms of this type of graph. Such manipulations do not reflect some deep economic insight, but are rather a strategy used to display relationships in this useful way.

7 Some Further Issues

We have now provided a careful statement of the partial-equilibrium demand-supply model familiar from an economic - principles course. While providing a review of this basic model, the primary purpose has been to illustrate the generic structure of economic models and to familiarize the reader with the terminology of these models. Two interrelated questions remain. First, how do we evaluate this model? Is it a "good" model, and what would we mean by that? Second, are there questions not adequately addressed by this model that suggest the need for a more refined model?

7.1 Evaluation of Models

What makes a "good" model? One answer to this question is given by how close a model hews to the five "epistemic virtues" that were spelled out in Chapter Two: predictive ability, internal coherence and external consistency, unification, fertility, and simplicity. A less encompassing but still useful answer is that a good model also helps us understand the problem at hand. At a very preliminary stage of investigation of a problem, a good model can be one that simply helps us organize thought about the problem. For many questions, the simple demand-supply model just presented is a "good" model.

In particular, consider what our model tells us about the effects on sugar production in the European Union and in the rest of the world that would arise from a reduction of the European Union sugar production subsidy. The answers given by our model are that EU production would fall and ROW production would increase. These answers are the keys to understanding parts of the political fights over this proposed policy. Many developing countries such as Brazil feel that they will benefit from the higher world sugar price and concomitant increase in Brazilian sugar production. Hence they have lobbied for such a change. European Union sugar producers, on the other hand, lobby against such a proposed change.

7.2 Unanswered Questions

For some of the most interesting questions of international trade, though, this model is inadequate. For one thing, what is needed to grapple with the idea of what economists mean by "gains from trade" is a model that looks behind the demand and supply curves and provides a fuller description of what is meant by tastes and technology, and how these notions constrain behavior. For another, much of the subject matter of international economics requires an understanding of the simultaneous interactions between the parts of all of the markets of an economy. Such a **general equilibrium** model has two important advantages over partial - equilibrium models for the purposes of understanding international trade issues. First, some of the most prominent arguments about the effects of trade policy concerns questions about whether jobs are created or destroyed. Remember, for example, the discussions in the introductory chapter about the loss of jobs in the textile industry arising from rising foreign imports, or the expected loss of jobs in "big steel" unless tariffs were imposed. General equilibrium models emphasize and make clear that on net, jobs are not "lost" in response to changing economic conditions, but are redistributed from one sector to another. Once one develops the habit of thinking in terms of general equilibrium, one quickly recognizes the less-emphasized related effects of job creation in other sectors whenever the "jobs" discussions about trade policy is broached. This is not to say that there are not real problems and hardships associated with the economic dislocations brought about by participation in the international economy, but rather that there are also opportunities not as readily apparent to someone not trained in general - equilibrium thinking.

Such general - equilibrium habits of thought also help one bring to mind more quickly analogous situations that help one think about trade policy effects. For example, other kinds of economic changes such as changes in technology have the same types of dislocation effects as does international trade. The development of word processors caused a loss of jobs in the typewriter manufacturing industry, but led to an expansion of a variety of other industries. This situation is analogous to the "textile" question in that both of these situations had the same generic effects on employment: jobs in some industries were lost, and jobs in other industries were gained.

The jobs issue is just one example of a general phenomenon highlighted by general-equilibrium models: the substitutability of resources within an economy. Recall from the first chapter the discussion about the popular view of "price gouging" by hoteliers in the research triangle area of North Carolina in anticipation of soon-to-be-held special Olympics. This view, we argued, was flawed because it failed to understand the ability of producers in an economy to substitute resources in the production of goods and services, and the ability of consumers in an economy to substitute one good for another in the face of changing incentives. This feature of *substitutability*, as one of the few key elements of the way economist's think about the world, is really only fully understood and appreciated within the context of a general equilibrium model.

We now move on to develop three key models for an understanding of international economics. First, we develop the simplest possible general equilibrium model we can envisage, one in which there are no production decisions. Such an economy, populated only by consumers but not producers, allows us to lay bare the basic novel features of general equilibrium models and some key features of international trade models. It also allows us to introduce the concept of "gains from trade" in its most pure and uncomplicated form.

Second, we introduce production into the general equilibrium framework. Because so much of international economics concerns the effects of changes in the mix of products produced by the factors of production available to the economy as a whole, knowledge of this model is essential to an understanding of both international trade and open-economy macroeconomics.

Finally, we analyze the demand and supply of money. Knowledge of how economists model this feature of the modern economy permits us to understand the key distinction between real and nominal variable.

16247 words

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