

The Wasserstein Gradient Flow of the Fisher Information and the Quantum Drift-Diffusion Equation

Ugo Gianazza

September 12, 2007

Abstract

I will present a global existence result for nonnegative variational solutions to the “quantum drift diffusion” evolution equation

$$\partial_t u + \operatorname{div} \left(u \left(2D \frac{\Delta \sqrt{u}}{\sqrt{u}} - f \right) \right) = 0$$

under variational boundary condition. Despite the lack of a maximum principle for fourth order equations, nonnegative solutions can be obtained as a limit of a variational approximation scheme by exploiting the particular structure of this equation, which is the gradient flow of the (perturbed) Fisher Information functional with respect to the Wasserstein distance between probability measures.

I will also discuss long time behaviour of the solutions, proving their exponential decay to the equilibrium state $g = e^{-V}$ characterized by

$$-\Delta V + \frac{1}{2} |DV|^2 = f, \quad \int e^{-V} dx = \int u_0 dx,$$

when the potential V is uniformly convex. This is a joint work with G. Savaré and G. Toscani.