Readiness for College Mathematics: Longitudinal Predictors Among Marginalized Students

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Abstract

The goal of the current study was to evaluate a range of predictors of scores on a college-readiness assessment for mathematics among students from marginalized backgrounds, informed by an integrated theory of mathematics achievement proposed by Watts and colleagues (2015). We evaluated preschool and middle-school predictors of math ACT scores in a longitudinal study of predominantly Black students from economically-disadvantaged homes in the United States (n = 329; 80% Black, 59% female). Students' mathematics knowledge at age 4 predicted their math ACT scores in 12th grade (B = .68), controlling for cognitive skills at age 4 and demographic characteristics. Their middle-school mathematics knowledge (number, algebra, and geometry knowledge; B = .43 - .67), and school placement (high-school math course in 8th grade, B = 1.5) were also unique predictors and mediated the relation between mathematics knowledge at age 4 and math ACT scores. In line with a fade-out model (Stipek & Valentino, 2015), executive function skill in 6th grade was not a unique predictor with prior mathematics knowledge in the model. In line with attitude-achievement paradoxes for Black students, mathematics self-concept in 6th grade was also not a unique predictor. Findings highlight the importance of simultaneously considering multiple factors and systems that influence students' mathematics achievement across preschool to 12th grade.

Readiness for College Mathematics: Longitudinal Predictors Among Marginalized Students

Objective

The goal of this study was to evaluate a range of predictors of math achievement in high-school among students from marginalized backgrounds. We used scores on the math section of the ACT as our outcome measure because the ACT is a widely-used standardized test in the United States that provides an overview of high-school students' preparedness for college coursework (ACT, 2022). Taking a college-readiness assessment such as the ACT is a high-school graduation requirement in about half of U.S. states, including where this study was conducted. Most past longitudinal research we reviewed focused on predicting math achievement using norm-referenced assessments administered by researchers. Student effort and motivation may be higher when taking the ACT, which has potential consequences for their high-school graduation and future educational options. Further, researcher-designed measures typically do not have evidence that scores are predictive of college readiness, whereas ACT math scores have strong predictive validity for successful performance in entry-level courses in college math, grade-point average in the first-year of college, and college completion (ACT, 2022).

Theoretical Framework: An Integrated Theory of Mathematics Achievement

Integrated theories are needed that consider a broad range of factors that influence math achievement. Theory and research on math development and achievement often focus on one type of factor, such as motivation, executive function, or school placement (e.g., Arens et al., 2017; Bull & Lee, 2014; Lee & Mao, 2021). Although useful, such an approach does not give a comprehensive account of the range of cognitive, academic, motivational and contextual influences on math achievement. Based on a review of the literature, Watts and colleagues (2015) identified four classes of factors with strong theoretical and empirical support for impacting math achievement and proposed them as central to an integrated theory of math achievement, illustrated in Figure 1. Three are individual, child-level factors, and the fourth factor is the school and classroom environment. These four types of factors fit within a broad bioecological systems perspective on development that considers both personal and environmental characteristics simultaneously (Bronfenbrenner & Morris, 2007).

To test their model, Watts and colleagues (2015) used data from the NICHD Study of Early Childcare and Youth Development (SECYD), conducted with a U.S. national sample of predominantly White, middle-class families. Their study focused on the factors that mediated the relation between math achievement at age 7 and 15, as measured at both ages with the Applied Problems subtest of the Woodcock-Johnson (Woodcock et al., 2001). Their findings are integrated into the sections below. In the current study, we tested the generalizability of their findings by considering (a) different operationalizations and measures of the contributing factors, (b) a very different sample - economically-disadvantaged students who were predominantly Black and living in a metropolitan area of the Southeastern United States, (c) an outcome with real-world implications, the ACT, and (d) a longer time span (age 4 to 18).

Prior Math Knowledge

First, consider the importance of prior math knowledge. Informal math knowledge develops even without formal instruction and includes foundational knowledge of numbers, patterning, measurement and shapes (Author, 2017). Children’s informal math knowledge near the beginning of formal schooling is a robust predictor of their math achievement in elementary school (Duncan et al., 2007; Jordan et al., 2009), with some evidence that it also predicts math achievement in high school, at least for White, middle-class students (Watts et al., 2014).

Given the hierarchical nature of math knowledge, middle-grades math knowledge should also predict math achievement in high school and mediate the relation between early and later math knowledge. For example, in Watts and colleagues (2015), fraction and division knowledge in fifth grade helped mediate the relation between first-grade and age 15 math achievement. In the current study, we considered three components of math knowledge in middle school that are emphasized in state math standards (e.g., Common Core State Standards, 2010) and on college-readiness assessments because they are needed for successful performance in entry-level courses in math in college and career training programs (ACT, 2022). First, there is a heavy emphasis on algebra and functions knowledge, including knowledge of expressions, equations, inequalities and functions (referred to as *algebra* for brevity). Second, geometry is another core topic and includes knowledge of shapes and solids, such as congruence and similarity relationships. Third, number and quantity knowledge, especially knowledge of the number system and of fractions, is another core topic and foundational to math achievement in high school (see also Siegler et al., 2011, 2012). Statistics and probability is a fourth core topic, but a measure of this knowledge was not available. Overall, we hypothesized that students' math knowledge in preschool and knowledge of algebra, geometry, and number in middle school would predict their math achievement in high school and that their middle-school math knowledge would help mediate the relation between early math knowledge and math achievement in high school.

Executive Function

Next, consider domain-general cognitive skills, particularly executive function skills. Executive function (*EF*) captures mental capacities to maintain and manipulate information (working memory), inhibit impulsive or incorrect responses (inhibition), and flexibly apply information in different ways (cognitive flexibility or shifting) (Miyake & Friedman, 2012). EF skills develop rapidly during early childhood and continue to improve across childhood and adolescence (Diamond, 2013). Math learning and problem solving rely on a variety of EF skills (see Bull & Lee, 2014 for a review). For example, math problem solving involves working memory, as the learner retrieves relevant knowledge from long-term memory and mentally manipulates information to find a solution. Spatial working memory is particularly important as the learner creates and updates a mental representation of a math problem while executing multiple solution steps (LeFevre et al., 2010). In addition, learners must inhibit use of incorrect, inefficient and inappropriate strategies or use of irrelevant information. They also must flexibly choose between alternative solution strategies and shift between different operations or steps of complex solution strategies.

Indeed, students with stronger EF skills also have stronger math knowledge concurrently and over time (Bull & Lee, 2014; Clements et al., 2016). For example, EF skills were concurrent predictors of 6th grade math achievement (Wilkey et al., 2018). Relations between EF skills and math knowledge can be bi-directional, perhaps because math learning is a context for practicing EF skills (Clements et al., 2016).

However, some theory and mounting evidence support a ‘fade-out' model, where individual differences in EF skills become less relevant in upper elementary school, compared to earlier grades, as differences in domain knowledge play a much stronger role in supporting learning (Stipek & Valentino, 2015). Indeed, multiple EF measures at age 5 predicted students' math knowledge at age 5 and growth in math knowledge from age 5 to 14, but the influence of these EF skills was much stronger in the early elementary grades than the later grades (Stipek & Valentino, 2015). Watts et al. (2015) reported that EF skills in grades 3-4 were not predictive of age 15 math achievement over and above first-grade math knowledge and other factors. In fact, even at early ages, prior math knowledge can be substantially more predictive than EF skills. EF skill at age 4.5 was predictive of age 15 math achievement on its own, but was no longer predictive after controlling for math knowledge at age 4.5 (Ahmed et al., 2019). All three of these studies used data from the NICHD SECCYD. We hypothesized that these findings with predominantly White, middle-class students would generalize to our sample, such that students' EF skills in middle school would not predict their math achievement in high school after controlling for their preschool math knowledge.

Self-Concept of Math Ability

Third, consider motivation, given that math learning and problem solving requires sustained, intentional effort. In particular, students are motivated to achieve in academic areas in which they expect to succeed, and students' self-concept of domain-specific ability captures their perceived academic capabilities and likelihood of succeeding in a domain (Eccles & Wigfield, 2002). This term is closely related to and often used interchangeably with the terms "expectancies of success," "self-efficacy," and "confidence" (Cook & Artino, 2016), in part because of difficulties in distinguishing between the constructs empirically (Eccles & Wigfield, 2002). Students with a more positive self-concept are more likely to actively and positively engage with math lessons and to take more advanced math classes (Guo et al., 2015; Schnitzler et al., 2021), which then influences their future self-concept (Marsh & Craven, 2006).

Decades of research has found that students' with a more positive math self-concept have higher future math achievement (Arens et al., 2017; Liu et al., 2022; Marsh & Craven, 2006). Relations are often reciprocal, with students' math achievement predicting their later math self-concept (Arens et al., 2017). For example, in a sample of White, high-achieving, college-bound students, students' self-concept of math ability in 10th grade was positively predictive of their future math SAT scores (Casey et al., 1997). Similarly, Black students' general academic self-concept positively predicted their concurrent math SAT scores (Payne, 1992). In Watts and colleagues (2015), more positive math self-concept in 6th grade was predictive of higher math achievement at age 15 and helped to explain the relation between students' first-grade math knowledge and later achievement.

Despite the general positive associations, the relation may differ for different groups of students. First, math self-concept is not consistently related to concurrent math achievement for Black high-school students, with weak, non-significant or even negative relations between the two in some cases (Cheema & Kitsantas, 2014; Kotok, 2017; Seo et al., 2019; Strayhorn, 2010). Positive attitudes toward education coupled with poor academic achievement have been labeled attitude-achievement paradoxes (Mickelson, 1990). As an example, in a representative sample of U.S. high-school students, Black and Latine students tended to have a positive math self-concept but relatively low math achievement, and grade 10 self-concept was less predictive of grade 12 achievement among Black and Latine students compared to White students (Seo et al., 2019). The Black and Latine students also reported, on average, greater value in schoolwork than their White peers, suggesting that devaluation of education was not a mechanism driving this paradox. Although the causes of this paradox are not well understood, it may reflect barriers to challenging learning opportunities and/or resistance to the marginalization Black and Latine students face in school and in society (Seo et al., 2019).

In addition, secondary-school students with lower achievement scores had a weaker concurrent relation between math self-concept and math achievement compared to students with higher achievement scores in three large-scale international studies (Keller et al., 2021). This was hypothesized to occur in part because students with lower achievement scores may have worked to defend against negative self-views (Alicke & Sedikides, 2009). In contrast, some evidence suggests positive, reciprocal, longitudinal relations between math achievement and self-concepts across low-, middle-, and high-achievement schooling tracks in Germany (Arens et al., 2017). Students in the current study, on average, were performing 2 years behind their actual age on standardized math assessments when in middle school (e.g., Authors, 2017, 2019). Because students in our study were predominantly Black and students with lower achievement scores in math, we hypothesized that students' math self-concept in middle school would not predict their math achievement in high school in our sample.

School Placement

Fourth, moving beyond child-level factors, the school and classroom environments influence students' math achievement. We focused on advanced academic opportunities at school, as they can provide rich opportunities for students to learn more advanced content. Watts and colleagues (2015) theorized that early math knowledge would influence the chance of placement into advanced academic programming, which in turn would expose students to higher quality instruction and/or to peers who were more academically motivated and capable. They found that placement into gifted and talented programs in elementary school was predictive of students' high-school math achievement and helped mediate the relation between first-grade math knowledge and later math achievement. In contrast, they found that placement into special-education programming was not a predictor or mediator of their high-school math achievement. As noted by Watts and colleagues (2015), the quality of classroom instruction and teacher characteristics are also important elements of the school and classroom environment, but measures of these specific factors were not available in their study or in the current study.

We considered another common advanced academic opportunity - an accelerated math track, with placement into a high-school math course in middle school. Approximately 24% of eighth graders in the U.S. were enrolled in a high-school math course (typically Algebra I) in 2015, with 80% of eighth graders having access to this option (U.S. Department of Education, 2018). However, nationwide, Black eighth graders were much less likely to be enrolled in this advanced math track (e.g., 12% of these students, compared to 24% of White students; U.S. Department of Education, 2018). This placement matters, as students who were placed into a high-school math course by the eighth grade had higher math achievement in high school and were more likely to enroll in calculus, even when using robust statistical modeling to examine selection bias and heterogeneity of effects (Lee & Mao, 2021; Rickles, 2013). In turn, enrollment in precalculus or calculus in high school was predictive of whether students attended college, especially a selective college or university (Byun et al., 2015). Thus, we hypothesized that placement into a high-school math course by eighth grade would be a positive predictor and mediator of their math achievement in high school.

Rather than a traditional Algebra I-Geometry-Algebra II high-school math sequence, many school districts in the U.S., including the school district in the current study, have adopted an Integrated Mathematics curriculum (Will, 2014). This approach is designed to integrate concepts from algebra and geometry every year for three years. Students enrolled in Integrated Mathematics courses demonstrated greater math knowledge than students enrolled in the traditional course sequence (e.g., Grouws et al., 2013), particularly in schools serving many students living in poverty (Krupa & Confrey, 2017).

Acknowledging Power and Privilege in the U.S. School System

A bioecological systems perspective highlights the importance of considering indirect influences on children's environment, including systemic, structural factors (Bronfenbrenner & Morris, 2007). This includes acknowledging the impact of racism, prejudice, discrimination, oppression and segregation on minoritized children and families (García Coll et al., 1996). When discussing racial groups, we must also remember that race is a social construct, not a biological distinction.

In the context of schooling, the U.S. education system has systematically advantaged White students with resources and power, and discriminatory policies and practices have prevented Black, Latine, and Indigenous children from becoming equal participants in schools (Ladson-Billings & Tate, 1995; Martin et al., 2017). This marginalization permeates the classroom—students of color attend schools on average with fewer resources and less qualified teachers, and they experience racial stereotyping (Ladson-Billings & Tate, 1995; Stinson, 2013; United States Government Accountability Office, 2022). Black students, in particular, are often positioned as less capable in math and in school (Nasir & Shah, 2011). These macrosystem-level factors provide important context for understanding potential differences by race and socioeconomic status described in previous sections. They also illustrate the importance of evaluating if findings from Watts and colleagues (2015) generalize to marginalized students, such as our sample of predominantly Black students living in homes with low family income and limited educational attainment. Our hypotheses are outlined in Figure 2.

Method

Participants

Participants were students drawn from a larger, longitudinal study on math development. An initial sample of 771 children were recruited in 2006 from 57 pre-k classrooms in a large urban city in the Southeastern United States, all of which served students who qualified for free or reduced priced lunch (family income < 1.85 times the U.S. federal income poverty guideline); less than 10% of the consented sample were English Language Learners. See Hofer and colleagues (2013) for more information on site selection and participant recruitment. In 2013, 628 parents or guardians could be located and 83% (n = 519) consented for their child to participate in this longitudinal follow-up study on math development (Vanderbilt Institutional Review Board #131804).

The current study included the 329 students who had ACT scores in school records from the participating school district. ACT scores were not available for 147 students who no longer attended school in the participating district in 12th grade or occasionally because they had a guardianship change that meant we no longer had parental consent. All students in the participating school district were expected to take the ACT at school for free as a graduation requirement, although scores were not available for 43 students. Most students graduated from high school in 2021, when they were 18 or 19 years old, but 18% had been retained at some point (10% before 2nd grade, 8% after 2nd grade) and graduated in 2022, and thus were 19 or 20 years old. The analytic sample (59% female) was 80% Black, 6% White, 9% non-White Hispanic, and 5% another race, including mixed race, according to school records, which was very similar to the original full sample (e.g., 55% female and 77% Black).

In high school, most of the students continued to live in economically-disadvantaged homes. Based on parental report when students were in 10th grade, and excluding 54 missing responses, the highest education level of a parent was less than a high school degree for 10% of students, a high school diploma or GED (alternative to high school diploma) for 56%, an Associate's degree for 16%, and at least a Bachelor's degree for 18% of students. Excluding the 74 students with missing parental income data, 31% of students came from families with annual household income less than $20,000, 26% came from families with incomes between $20,000 and $34,999, 21% came from families with between $35,000 and $49,999, and 22% came from families with an annual household income of $50,000 or higher. In comparison, the median national annual household income was $69,560 in 2019 (Shrider et al., 2021). Students were distributed across 28 public high schools, most serving primarily Black and Latine students and students from economically disadvantaged homes. A power analysis in Optimal Design for the two outcome measures indicated that our sample size of 329 had 99% power to detect a medium effect size ($d = .5$) and 75-78% power to detect a medium-small effect size ($d = .3$) in multilevel models with 10 predictor variables and 7 control variables.

Measures

Outcome

We used students' scores on the math section of the ACT as our outcome. The math section has 60 questions that cover algebra, geometry, functions, statistics and probability, and includes complex questions that require integration. ACT scores were taken from school records, and the most recent ACT score was used if a student had taken the ACT multiple times. According to the ACT (2022), reliability of math scores is high ($\alpha = .92$).

Prior Math Knowledge

Preschool Math Knowledge. Three assessments of preschool math knowledge were used. The Quantitative Concepts subtest of the Woodcock Johnson Achievement Battery III assessed students' knowledge of basic numerical concepts, symbols, and vocabulary, including numerical sequences, and the Applied Problems subtest assessed students' ability to analyze and solve oral math problems (Schrank et al., 2001). According to its technical manual, the Quantitative Concepts and Applied Problems' median test reliability was .91 and .94 respectively. (Schrank et al., 2001). The Research-Based Early Mathematics Assessment (Clements et al., 2008) assessed numeracy knowledge as well as geometry, patterning, and measurement knowledge, and item reliability was .94 and inter-rater reliability was .98 in its validation studies. A composite measure of math achievement was created by transforming raw achievement scores on the three subtests into z-scores, and then taking the mean of the z-scores.

Middle-school Math Knowledge. Math knowledge in 6th grade was assessed using subtests of the KeyMath 3 Diagnostic Assessment (Connolly, 2007) as well as the Quantitative Concepts subtest from the Woodcock Johnson Achievement Battery III. The algebra subtest of KeyMath measures students' understanding of pre-algebraic and algebraic concepts, including recognizing and describing patterns and functions; working with operational properties, variables, and equations; and representing mathematical relations. The geometry subtest measures students' spatial reasoning and ability to analyze, describe, compare, and classify two- and three-dimensional shapes. We used their age-scaled scores for both subtests. The numeration subtest was also administered, but we did not use these scores because they were highly correlated (>.8) with algebra scores. According to its manual, the internal-consistency reliability for the algebra and geometry subtests ranged from .79 to .90 for 6th grade, and the test-retest reliability ranged from .78 to .85 for pre-kindergarten to 12th grade. The Quantitative Concepts subtest was used as a measure of number knowledge. We used W scores.

Math Self-Concept

Math self-concept in sixth grade was measured using nine items from the 2011 Trends in International Mathematics and Science Study (TIMSS) Math Attitudes Questionnaire, self-concept subscale (Mullis et al., 2012), and reliability was good, α = .87. The scale used a 1 to 4 Likert scale, from agree a lot (1) to disagree a lot (4). Example items included: "I am good at working out hard math problems" and "My teacher thinks I can do well in math classes with difficult materials." See Appendix for all items. Items were re-coded so that a higher score corresponded to higher motivation.

Executive Function (EF)

The Hearts and Flowers task was used to measure inhibition and cognitive flexibility (Davidson et al., 2006; Diamond, 2013). This task involves selecting the corresponding key on a keyboard based on the display of a heart or a flower. In the first block, students are instructed to select the key that corresponds to the side of the screen the heart appears on. In the second block, students are instructed to select the key that corresponds to the *opposite* side of the screen the flower appears on. These two blocks both have 12 trials. The third block (48 trials) are mixed trials, and both hearts and flowers appear individually. Mean accuracy for the mixed trials was used as an indicator of cognitive flexibility, with evidence from past research of sufficient reliability (α = .73) and validity (correlated with other measures of executive function .25-.33; Traverso et al., 2020). The average difference in response time between flower trials and heart trials was used as an indicator of inhibitory control (Diamond, 2013).

The backward Corsi block-tapping test was used as a measure of visual–spatial working memory (Corsi, 1972; Kessels et al., 2000). This computerized task involves students watching a sequence of squares lighting up. Then, students are asked to tap the squares in reverse viewing order. There are two practice trials. For the 14 non-practice trials, students are given two attempts per trial, and if at least one attempt is correct, they move on to the next longer and more difficult trial. The sequences increase from 2 squares, up to 8 squares. We use the longest span in a correctly repeated sequence as the score of interest, in line with past research (e.g., Pollack et al., 2022). Its internal consistency has ranged from .70 to .73 (Orsini, 1994). If students did not have usable data or did not complete the Corsi in 6th grade, we used their standardized score from 5th grade. The two Hearts and Flowers measures and Corsi were standardized, and a composite EF variable was created by averaging the three standardized scores.

Accelerated Math Course Placement

Whether students were enrolled in a high-school math course in 8th grade was coded from school records as a binary variable. Most schools offered Integrated Mathematics I, but a few charter schools offered Algebra I.

Other Early Domain General Cognitive and Academic Skills

Attention and Self-Regulated Behavior (Preschool). A direct measure of EF was not administered in preschool, but two related variables were available and were included as control variables. Attentive behavior was measured via teachers' ratings on work-related skill items on the Cooper–Farran Behavioral Rating Scale (Cooper & Farran, 1991). These 16 items assessed students' attentiveness, ability to follow directions, and task persistence in the classroom on a 7-point behavior anchored scale, and reliability was strong in this sample (α = .95).

Self-regulated behavior includes the ability to plan and finish tasks and was measured via teachers' ratings on 4 relevant items on the Instrumental Competence Scale for Young Children-Short Form (Lange & Adler, 1997). Teachers rated behavior as displayed in the classroom on a 4-point Likert scale, and items included "finishes tasks and activities" and "has difficulty planning and carrying out activities that have several steps (reverse coded)." Scores could range from 1 to 4, with 4 reflecting the greatest regulation, and reliability was good in this sample ($\alpha > .84$). Because the two scores were highly correlated (> .8), scores were standardized, and a composite variable was created by averaging the standardized scores.

Narrative Recall (Preschool). Narrative recall was used as a direct measure of the combination of language skill, intelligence, and working memory capacity in preschool, as narrative recall is correlated with measures of vocabulary, verbal IQ and working memory (Florit et al., 2009). Narrative recall was measured with the information score from the Renfrew Bus Story–North American Edition (Glasgow & Cowley, 1994), which scores the accuracy of students' retelling of a narrative. The Renfrew Bus Story developers indicate that the test-retest reliability of the information score is .79 (Hayward, Stewart, Phillips, Norris, & Lovell, 2008).

Reading (Preschool). Early reading skill is also predictive of math achievement (Duncan et al., 2007; Watts et al., 2014). As a measure of reading skill, the Letter–Word Identification subtest of the Woodcock Johnson III Tests of Achievement was used. It assesses students' letter and word identification ability. *W* scores were used. The reported median reliability coefficient for this test is .94, using a split-half procedure (Schrank et al., 2001).

Procedure

All direct measures of students' knowledge in preschool and middle-school were collected one-on-one in a quiet room at the students' schools by trained research assistants. For all direct measures, except the Renfrew Bus Story, there was a stop criterion such that after students failed a specified number of items (as specified by the implementation manual), the assessment was stopped. We used preschool measures that were collected at entry into the study in fall of 2006, at the beginning of the pre-k school year (mean age = 4.4, *SD* = .3). For our middle-school measures, we elected to use data collected when most students were finishing 6th grade, and all students, even those who had been retained a year, had transitioned to middle school (mean age = 12, *SD* = .3). The early measures were also administered near the end of pre-K through first grade (see Hofer et al., 2013), and the middle-school measures were administered when most students were near the end of grades 5-10, and additional cognitive and numerical cognition variables were administered in some years (see Pollack et al., 2022; Author, 2017; Wilkey et al., 2018). In middle school, students received $25 cash each year they completed our assessments. Most students took the ACT during the 2020-2021 school year, and due to the COVID-19 pandemic, high schools in the district (with the exception of a few charter schools) did not open for in-person instruction until March 2021; however, students were expected to come in-person to school to take the ACT, per state policy.

Data Analysis

This study's design and its analysis were not pre-registered. Some students were missing data at one or two of the time points. For early predictors, missing data was rare (1%). For middle school, no students were missing the measures of math knowledge, less than 1% were missing data for the EF and self-concept measures, and 3% were missing information on their math placement in 8th grade. More common was missing information on control variables, most often parental education level (16%) and parental income level (22%). We used multiple imputation in SPSS to impute missing predictor variables. We imputed 30 datasets and reported pooled results when feasible. For the mediation analysis using a bootstrapping method, we aggregated the imputed 30 datasets.

In all statistical models, we controlled for age, gender, race (Black or not), highest education level of a parent from 10th grade report, annual household income level in 10th grade and whether and when (before or after 2nd grade) students had been retained. All predictor variables were standardized so that estimates were easier to interpret. Because many correlations among predictors were moderate to high (but all below .8), we tested for multicollinearity by estimating variance inflation scores (VIF) for all independent variables. All VIF scores for independent variables were < 5.

Intraclass correlations by school attended in 12th grade were moderate (ICC = .15), so we used multilevel regression models with two levels: (a) the individual level and (b) the school level. We ran separate regression models with: 1) only early predictors and controls; 2) only middle-school predictors and controls; and 3) both time points together. We also conducted a mediation model to test if middle-school predictors mediated the relation between preschool math knowledge and ACT scores using the bootstrapping technique recommended by Preacher and Hayes (2008) to obtain estimates for the indirect effects of the preschool predictor(s).

Results

Table 1 presents descriptive statistics and correlations between predictors and ACT scores. Math ACT scores had significant positive bivariate and partial correlations with all predictor variables. Preschool math knowledge was also positively correlated with middle-school math knowledge and EF skills, but not math self-concept in middle school.

As hypothesized, preschool math knowledge, measured near entry into pre-k programs, positively predicted ACT math scores in 12th grade (B = .68 [.26, 1.1]), controlling for cognitive and other academic skills in preschool and background characteristics (see Table 2, model 1). For each standard deviation increase on the preschool math knowledge composite, math ACT scores were predicted to increase .68. None of the other early cognitive and academic variables were a significant predictor.

As hypothesized, each type of 6th grade math knowledge and accelerated math course placement predicted math ACT scores in 12th grade (see Table 2, model 2). Specifically, higher number (*B* = .43 [.09, .76]), algebra (*B* = .59 [.18, 1.0]), and geometry knowledge (*B* = .67 [.34, 1.0]) predicted higher math ACT scores. Students with an accelerated math course placement in 8th grade were predicted to score 1.5 points higher on the math ACT. As hypothesized, EF and math self-concept in 6th grade did not predict math ACT scores with other variables in the model.

Consistent with our mediation hypothesis, preschool math knowledge did not help predict math ACT scores with middle-school variables in the model (see Table 2, Model 3). Regression estimates for the middle-school knowledge and course placement variables decreased very little with preschool variables in the model.

As hypothesized, the bootstrap results for indirect effects of preschool math knowledge on ACT math scores indicated that all three middle-school math knowledge measures and accelerated math course placement mediated the effect of preschool math knowledge on math ACT scores (see Table 3).

Discussion

The ACT is a widely-used standardized test that provides an overview of high-school students' preparedness for college coursework. As hypothesized, in our sample of predominantly Black students living in homes with low family income and limited educational attainment, students' math knowledge in preschool was predictive of their math ACT scores 13 years later (see Figure 2). This relation was mediated through students' algebra, geometry and number knowledge in 6th grade and placement into an accelerated math course in 8th grade. In contrast, executive function skill and math self-concept in 6th grade were not unique predictors in this sample, as predicted.

Implications for Integrated Theories of Mathematics Achievement

Rather than focusing on only one or two types of factors that influence math development, integrated theories of math achievement are needed. We consider implications for the four factors proposed and evaluated by Watts et al. (2015) with a sample of predominantly White and middle-class students, and illustrated in Figure 1.

Prior Math Knowledge

The current study extends past research demonstrating that children's preschool math knowledge was predictive of their math achievement on a research-designed measure at age 15 among a sample of predominantly White, middle-class children (Watts et al., 2014). Thus, informal math knowledge developed outside of formal schooling can predict math achievement beyond elementary school and merits serious attention (Duncan et al., 2007; Jordan et al., 2009). For example, parents vary widely in how often and in what ways they engage their preschool-aged children in early math-learning experiences, and this variability is related to children's concurrent and later math knowledge (Douglas et al., 2021). Providing parents with support to engage in math activities at home, such as math story time, increased children's math achievement (Berkowitz, et al., 2015).

Middle-school math knowledge, particularly geometry, algebra and number knowledge, played an important mediating role in the link between preschool math knowledge and math achievement in high school. First consider middle-school geometry knowledge. This is the first study, to our knowledge, to provide evidence for the predictive role of geometry knowledge for later math achievement. The finding aligns with calls for greater attention to developing understanding of geometry in elementary and middle school (Clements & Sarama, 2011; Lehrer & Chazan, 1998). Further, it is the first evidence that preschool math knowledge, which is primarily numeracy knowledge, was predictive of middle-school geometry knowledge, illustrating the inter-relations between the different strands of math. Second, the findings provide further evidence of the importance of developing algebra knowledge before high school, in line with the increasing attention to algebraic thinking throughout schooling (Blanton et al., 2019; Hornburg et al., 2022). Third, the findings provide additional support for the importance of strong number knowledge earlier in schooling for high-school math achievement (Siegler et al., 2011, 2012) and college readiness in mathematics. Future research is needed on predictive and mediating roles of other aspects of math knowledge, including measurement knowledge (Szilágyi et al., 2013) and data and statistics knowledge (Cobb et al., 2003).

School and Classroom Environment

School placement can have substantial influence on students' environment, including their learning opportunities. Watts and colleagues (2015) found that placement in gifted and talented programming in elementary school, but not in special education programming, was predictive of students' math achievement in high school. We considered school placement in an advanced math track, and in particular, placement into a high-school math course in 8th grade.

The current finding aligns with past evidence that students who were placed into an Algebra 1 course by the eighth grade had higher math achievement in high school, even when using robust statistical modeling to examine selection bias and heterogeneity of effects (Lee &

Mao, 2021; Rickles, 2013) and extends this finding to an Integrated Mathematics course sequence. Given the potential benefits of accelerated placement into a high-school math course in middle school, there is an urgent need to address barriers for Black students' equal participation in this advanced academic opportunity, as these students are half as likely to be enrolled in this opportunity compared to White students (U.S. Department of Education, 2018). One barrier is that high-school math courses are less likely to be offered at urban and rural middle schools, compared to suburban schools (U.S. Department of Education, 2018). Another potential barrier is separating students into a general versus accelerated math track in elementary or middle school. Students in a general math track, compared to an accelerated track, tend to have less access to highly qualified math teachers and fewer classroom opportunities for active learning, potentially denying students opportunities to develop skills in elementary or middle school that should increase success in high-school courses (Darling-Hammond, 2004; Oakes, 1990).

Future research needs to integrate measures of the quality of classroom instruction and teacher qualifications as additional important measures of the school and classroom environment. For example, past research indicates that positive teacher-student relationships facilitate student achievement and motivation and may be particularly important for Black and Latine students (Eccles et al., 1993; Ferguson, 2002). Opportunities for student interaction and collaboration are also associated with higher math achievement and valuing of math (Wang, 2012). Teachers' math and pedagogical knowledge are also related to higher-quality teaching and greater student achievement (Hill et al., 2005).

Domain-General Skills

Students' domain-general skills can also influence how, and how well, students learn math. EF skills are thought to be particularly influential to learning early in math development (Bull & Lee, 2014). Our findings are consistent with a 'fade-out' model, where individual differences in EF skills become less relevant in upper elementary school and beyond, as differences in domain knowledge and skills play a much stronger role in supporting learning (Stipek & Valentino, 2015). Our findings support the generalization of previous findings of fade-out from studies conducted with primarily White, middle-class students (Ahmed et al., 2019; Geary et al., 2017; Stipek & Valentino, 2015; Watts et al., 2015). Alternatively, rather than a fade-out, EF skills may not play a substantial and unique role even early in math development when controlling for other factors, as fixed effects analyses and latent growth curve models suggest smaller and non-significant relations between EF skills and math knowledge in kindergarten through 2nd grade, including for students from economically disadvantaged homes (Willoughby et al., 2012, 2019). This is an important area of future research.

Other cognitive skills, such as nonverbal reasoning, language ability, reading skill, and processing speed, should also be considered in integrated theories of math development. Each has been a positive predictor of concurrent and future math achievement in late elementary and middle-school (Fuchs et al., 2014; Geary et al., 2017).

Motivation

Motivation is another important factor to consider, as math learning and problem solving requires sustained, intentional effort. Decades of research has found that a more positive math self-concept predicted better math achievement in the future, at least for middle-class, White students (e.g., Liu et al., 2022; Marsh & Craven, 2006). In contrast, students in the current study had a positive math self-concept, on average, despite generally low math achievement. Further, although positively correlated with ACT scores, math self-concept was not a unique predictor of math ACT scores. These findings are in line with other evidence for an attitude-achievement paradox among Black students reviewed in the introduction (Cheema & Kitsantas, 2014; Kotok, 2017; Seo et al., 2019; Strayhorn, 2010) and a close to zero relation between self-concept and achievement for students with lower achievement scores (Keller et al., 2021).

Seo and colleagues (2019) proposed several potential reasons for the paradox of positive self-concept and low academic achievement that has been reported for Black, as well as Latine, students. First, these students may face barriers that reduce the benefits of a positive math self-concept for engaging in activities that benefit future math achievement (e.g., discrimination and stereotype threat may reduce pursuit of more challenging learning opportunities). Second, self-concept is informed by comparison to peers, and because Black and Latine students are more likely to attend low-resourced schools with many low-achieving students, students at these school may have higher self-concepts than equally competent students attending well-resourced schools with many high-achieving students (Marsh & Hau, 2003). Third, cultural resources and strong ethnic and racial identity may support positive self-concepts among Black and Latine students despite lower achievement, as a form of resistance to social stigmas.

Alicke and Sedikides (2009) proposed a different set of reasons that students with lower achievement scores may work to defend against negative self-views, eliminating the association between academic self-concept and achievement. Being asked to evaluate one's own abilities in self-concept questionnaires may trigger self-protective strategies. Self-protection originates from the assumption that people want to feel good or avoid feeling bad about themselves.
Self-protection strategies may include changing the comparison group, having poorer recall of

negative feedback, or downplaying the importance of negative feedback. For example, students with poorer grades overestimated their grades from the previous semester more than students with better grades (Gramzow et al., 2003). Future research is needed to better understand the effects of self-protection strategies, structural barriers, and cultural resources in moderating the effect of math self-concept on future math achievement.

Second, motivation encompasses a broad range of factors beyond self-concept of ability, including interest and attainment value (Eccles & Wigfield, 2002). Students with greater interest in math might spontaneously attend to mathematical information in their environment more, actively and positively engage with math lessons, and/or take more advanced math classes. Indeed, in predominantly White samples, students' math interest was a unique predictor of their later math achievement (Kriegbaum et al., 2015; Murayama et al., 2013), and a concurrent predictor of their math SAT scores (Burrus & Moore, 2016). However, interest was not related to concurrent math achievement among high-achieving Black students, though there was a significant correlation for high-achieving White and Latine students in the same study (Andersen & Ward, 2014). Math attainment value (one's personal importance of doing well on a task) was negatively related to math underachievement (i.e., the discrepancy between math ability test scores and grades) in a racially diverse sample of high school students (Fong & Kremer, 2020).

Further, students who have a sense of belonging in academic settings are more motivated to engage and learn. In a recent study with a racially and ethnically diverse sample of U.S. public school students, sense of belonging in math was the only significant predictor of algebra learning from classroom instruction, while measures of motivation (i.e., self-concept, interest and importance) were not (Barbieri & Miller-Cotto, 2021). Further, sense of belonging in math was lower among students from racial and ethnic groups that are underrepresented in STEM careers compared to White and Asian students, and a lower sense of belonging helped account for why students from underrepresented groups learned less (Barbieri & Miller-Cotto, 2021). It can be difficult to see yourself as a "math person" if you receive little recognition from teachers (Cribbs et al., 2015) or feel little sense of belonging in math spaces (Dortch & Patel, 2017). Overall, integrated theories of math development need to identify particularly influential motivational constructs, and consider environmental, structural and cultural factors that might moderate the influence of motivation on achievement for marginalized students.

Expanding Integrated Theories of Mathematics Achievement

More broadly, integrated theories of math achievement need to be revised to more explicitly attend to indirect influences on children's environment, including systemic, structural factors (Bronfenbrenner & Morris, 2007). For example, racism, prejudice, discrimination, and oppression impact the development of minoritized children and families (García Coll et al., 1996). These macrosystem level forces shape experiences in math classrooms, with Black students positioned as less capable than White students (Martin et al., 2017; Nasir & Shah, 2011). This positioning may also limit Black students' access to advanced math courses. For example, school counselors were less likely to recommend students for AP Calculus when the students' name was a common Black girls' name, compared to a common male or White name (Francis et al., 2019). In addition, school segregation, both between and within schools (e.g., via tracking), continues to be common in the U.S., often with fewer resources and worse outcomes at schools serving primarily Black and Latinx students from economically disadvantaged families (United States Government Accountability Office, 2022). School segregation was likely experienced by many of our participating students. To fully understand math development, we need to better integrate macrosystem level influences on children's environment with personal and school level influences.

Practical Implications

Current findings also have practical implications. ACT scores are often considered in college admissions, and ACT math scores have strong predictive validity for successful performance in entry-level courses in college math, grade-point average in the first-year of college, and college completion (ACT, 2022). In addition, poor performance in math is a barrier to college education and numerous career paths, and this is especially true for racially- and economically-marginalized students (Lee, 2012). Thus, the predictive power of limited math knowledge at age 4 for poor ACT scores 13 years later is troubling. Meeting marginalized students' needs and providing them with the opportunities and support they deserve requires counteracting a range of factors at the micro-, exo- and macro-system levels.

Limitations

Many factors influence students' environment, and current findings may not generalize to marginalized students attending suburban or rural schools, those from middle- and upper-middle class families, or primarily high-achieving students. The COVID-19 pandemic is another factor to consider, as the pandemic may have had varying impacts on individual students' environment and their performance on the math ACT. It is possible that findings from this study will not generalize to other historical periods, highlighting the need to attend to the Chronosystem level of Bronfenbrenner's bioecological systems model (Bronfenbrenner & Morris, 2007). At the individual level, we did not have direct measures of students' executive function skills or motivation in preschool or elementary school, so we were not able to consider the influence of these two factors early in schooling.

Conclusion

Students' prior content knowledge and placement in advanced coursework are two consistent predictors of their math achievement across multiple studies involving students from varied racial and ethnic groups and varied social and economic classes. In contrast, students' executive function skill and math self-concept in middle school were not unique predictors in the current study and several past studies. Integrated theories of math achievement need to simultaneously consider child-level factors, such as prior knowledge, and environmental factors, such as course placement. They should also consider systems that indirectly influence children's environment.

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Appendix

Items comprising the TIMSS math attitudinal scale

Items reproduced from Mullis et al. (2012) with slight wording changes. Students rated items on a 4 point likert scale with from agree a lot (1) to disagree a lot (4). An asterisk (*) indicates an item that was reverse-coded to allow higher scores to correspond to more positive attitudes.

Students confident in math scale (Self-concept)

1. I usually do well in math*
2. Math is more difficult for me than for many of my classmates
3. Math is not one of my strengths
4. I learn things quickly in math*
5. Math makes me confused and nervous
6. I am good at working out difficult math problems*
7. My teacher thinks I can do well in math classes with difficult materials*
8. My teacher tells me that I am good at math*
9. Math is harder for me than any other subject

Table 1

Correlations between Math ACT Scores and Key Predictors

	M	SD	1	2	3	4	5	6	7	8	9	10
1. Math ACT	15.5	2.8	–	.33***	.17**	.20***	.23***	.48***	.54***	.52***	.22***	.24***
2. PS Math	.06	.86	.35***	—-	.48***	.56***	.48***	.32***	.32***	.39***	.07	.19***
3. PS Attention & Self-Reg.[a]	0	1	.22***	.56***	—	.34***	.28***	.13*	.11*	.19***	.09	.09
4. PS Reading	322.3	25.6	.22***	.64***	.42***	—	.26***	.19***	.14***	.17***	-.06	.03
5. PS Narrative Recall	8.5	5.7	.25***	.56***	.35***	.37***	—	.17**	.18***	.19***	-.02	.09
6. MS Number	507.6	12.9	.52***	.41***	.26***	.30***	.25***	—	.68***	.53***	.28***	.30***
7. MS Algebra	8.4	2.9	.58***	.34***	.19***	.20***	.22***	.70***	—	.68***	.41***	.38***
8. MS Geometry	7.9	2.3	.57***	.37***	.24***	.20***	.20***	.57***	.71***	—-	.31***	.39***
9. MS Self-concept	3.1	.67	.21***	.07	.10	-.04	-.02	.28***	.40***	.30***	—-	.22***
10. MS EF[a]	0	1	.26***	.22***	.12*	.05	.11*	.30***	.37***	.38***	.22***	—-

Notes. PS = preschool, MS = middle school, EF = executive function; [a]Composite variables created from standardized values; **p* < .05. ***p* < .01, ***p<.001. Bivariate correlations are below the diagonal; above the diagonal are partial correlations, partialling out age, sex, race, whether retained a grade level, and parents' education level and income level.

Table 2

Regression Estimates Predicting ACT Math Scores From Preschool and Middle-School Predictors

	Model 1: PS		Model 2: MS		Model 3: All	
	Est.	SE	Est.	SE	Est.	SE
PS Math knowledge	.68**	.21	—	—	.04	.20
PS Attention & Self-regulation	.12	.18	—	—	.07	.15
PS Reading	.01	.18	—	—	.12	.15
PS Narrative Recall	.20	.16	—	—	.19	.14
MS Number knowledge	—	—	.43*	.17	.39*	.17
MS Algebra knowledge	—	—	.59**	.21	.57**	.21
MS Geometry knowledge	—	—	.67***	.17	.61***	.18
MS Self-concept	—	—	-.05	.13	-.01	.13
MS Executive Function	—	—	-.21	.25	-.22	.25
MS Accel. course	—	—	1.5***	.41	1.46***	.41
Controls	Incl.		Incl.		Incl.	

Note. PS = preschool, MS = middle school, Incl. = all models presented include the full list of control variables: age, sex, race, whether and when retained a grade-level, parents' education level and income level in 10th grade. *$p < .05$. **$p < .01$. ***$p < .001$

Table 3

Bootstrap Results for Indirect Effects of Preschool Math Knowledge on ACT Math Scores Through Proposed Mediators in Middle School

	Lower	Upper
Number knowledge	.02	.58
Algebra knowledge	.03	.60
Geometry knowledge	.15	.47
Accelerated course	.02	.52
Total Indirect	.54	1.6
Controls	Incl.	Incl.

Note. Values represent 95% bias corrected confidence intervals. All confidence intervals that exclude zero are significant at $p < .05$. Incl. = all models presented include the full list of control variables: age, sex, race, whether and when retained a grade-level, parents' education level and income level in 10th grade.

Figure 1

An Integrated Theory of Mathematics Achievement, based on Watts and colleagues (2015)

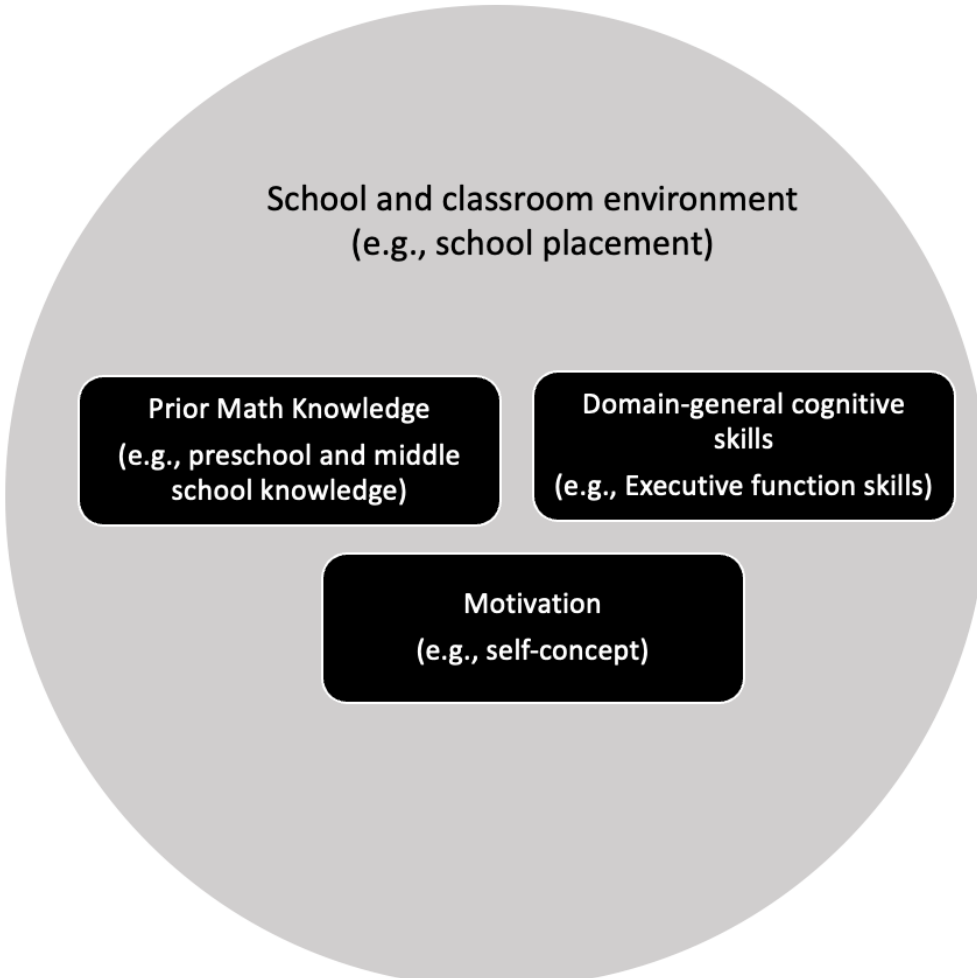

Figure 2

Predictions from an Integrated Theory of Mathematics Achievement for Current Study with Marginalized Students