Title

Cognitive Mechanisms Underlying the Relation Between Nonsymbolic and Symbolic Magnitude Processing and their Relation to Math.

Authors

Gavin R. Price[1], Eric D. Wilkey[1].

Author Affiliations

[1] Department of Psychology & Human Development, Peabody College, Vanderbilt University, 230 Appleton Place, Nashville, TN, 37203.

Corresponding Author:

Gavin R. Price

Email: gavin.price@vanderbilt.edu

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ABSTRACT

Recent studies suggest that the relation between nonsymbolic magnitude processing skills and math competence is mediated by symbolic number processing. However, less is known about whether mapping between nonsymbolic and symbolic magnitude representations also mediates that relation, and whether the mediating role of symbolic number processing is explained by domain general executive functions. Therefore, the current study examines whether symbolic comparison, mixed-format comparison, executive function, and visuo-spatial working memory each mediate the relation between nonsymbolic magnitude processing and math. Furthermore, we investigate whether the relation between nonsymbolic and symbolic magnitude comparison is mediated by mapping between the formats and/or domain general executive functions. Results indicate that symbolic processing mediates the relation between nonsymbolic and math, even after controlling for executive function and visuo-spatial working memory, which were also significant mediators. Cross-format comparison (i.e. mapping), on the other hand, did not mediate the relation between nonsymbolic comparison and math, but did mediate the relation between nonsymbolic and symbolic magnitude processing, even after controlling for executive function and visuo-spatial working memory, which also mediated that relation. Taken together, our results suggest that both domain-specific and domain-general cognitive mechanisms account for the link between nonsymbolic and symbolic magnitude processing and their relation to math.

INTRODUCTION

Mathematical competence is an important predictor of success in modern life, including educational achievement, employment, financial stability, and physical and mental health (Bynner & Parsons, 1997; Gross, Hudson, & Price, 2009; Parsons & Bynner, 2005) . However, a large number of individuals fail to acquire the math skills necessary to function optimally in today's society (Gross et al., 2009; NCES, 2007). Over the past decade, a growing body of research has elucidated important links between basic numerical processing abilities and the development of school level mathematical skills. In particular, it has been suggested that the ability to efficiently process numerical magnitude information in both nonsymbolic (e.g. sets of dots) and symbolic (e.g. Arabic digits) formats is an important foundational competence for math development (for a review see De Smedt, Noël, Gilmore, & Ansari, 2013). Nonsymbolic magnitude processing is typically measured using tasks that require participants to judge which of two sets of dots or other objects contains more items. Performance on this task has been suggested to reflect the precision of the so-called 'approximate number system' (ANS) (Feigenson, Dehaene, & Spelke, 2004). Nonsymbolic magnitude comparison performance has been shown to predict math competence in typically developing children and adults (Halberda, Mazzocco, & Feigenson, 2008; Libertus, Odic, & Halberda, 2012; Mazzocco, Feigenson, & Halberda, 2011b) and to be impaired in children with mathematical learning difficulties (Mazzocco, Feigenson, & Halberda, 2011a; M Piazza et al., 2010). It should also be noted, however, that a number of studies have tested for and not observed a significant relation between nonsymbolic magnitude comparison and math performance in both children and adults (e.g. Holloway & Ansari, 2009; Mundy & Gilmore, 2009; Price, Palmer, Battista, & Ansari, 2012). At the same time, a number of studies have reported significant relations between symbolic magnitude comparison tasks, in which participants compare the relative numerical size of two Arabic digits, and math competence (e.g. Bugden & Ansari, 2011; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009). However, again it should be noted that some studies have tested for and not observed any such relation (e.g. Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Therefore, as an alternative to contrasting the independent relations between nonsymbolic and symbolic magnitude processing and math competence, it may be fruitful to consider the developmental interplay between them. Specifically, recent evidence suggests that the relation between nonsymbolic magnitude processing and math may be mediated by symbolic magnitude and ordinality processing (Fazio, Bailey, Thompson, & Siegler, 2014; Lyons & Beilock, 2011; Price & Fuchs, 2016), numeral knowledge (Peng, Yang, & Meng, 2017) and 'number-numerosity mapping' as indexed by dot set estimation (Wong, Ho, & Tang, 2016). According to these studies, nonsymbolic magnitude processing may influence math outcomes by facilitating the acquisition of numerical symbols, which in turn influences the acquisition of basic math skills. However, while both Lyons & Beilock (2011) and Wong et al (2016) controlled for working memory in their models, it is unclear whether the relation between nonsymbolic magnitude processing and math is also mediated by domain general mechanisms of executive function and visuo-spatial working memory. Recent studies have shown that the relation between nonsymbolic magnitude processing and math performance is non-significant when controlling for inhibitory control (Fuhs & McNeil, 2013; Gilmore et al., 2013). Furthermore, the brain mechanisms in the frontal and parietal lobes, thought to support both symbolic and nonsymbolic magnitude processing, are also engaged during inhibitory control and working memory tasks (e.g. Banich & Depue, 2015; Dumontheil & Klingberg, 2011; Durston et al., 2002; Olesen, Westerberg, & Klingberg, 2004), suggesting that their activation during numerical processing tasks may not reflect solely domain-specific numerical magnitude processing. Therefore, in the current study we examine whether symbolic comparison, mixed-format comparison, executive function, and visuo-spatial working memory each mediate the relation between nonsymbolic magnitude processing and math, and whether any mediating role of symbolic number processing is accounted for by executive function and working memory.

A growing body of evidence also suggests that there may be a bidirectional influence between nonsymbolic and symbolic magnitude processing whereby the acquisition of numerical symbols refines the representation of nonsymbolic magnitude (Mussolin, Nys, Leybaert, & Content, 2015; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). Therefore, we also examine whether nonsymbolic comparison, mixed-format comparison (i.e. simultaneous symbolic and nonsymbolic formats), executive function, and visuo-spatial working memory each mediate the relation between symbolic magnitude processing and math.

The apparent importance of the relation between nonsymbolic and symbolic magnitude processing and math development gives rise to a second important question. Specifically, what are the cognitive mechanisms underlying the relation between Arabic digits and the quantities they represent? The most prominent current theory, the ‘mapping hypothesis’, suggests that Arabic digits are associated with or ‘mapped onto’ the innate ANS over the course of learning (Dehaene, 2007; Manuela Piazza, 2011; for a review see Leibovich & Ansari, 2016). Evidence for this theory comes largely from the fact that across studies, number comparison tasks using both nonsymbolic and symbolic stimuli demonstrate numerical ratio effects, whereby comparison performance declines as the ratio of the larger to the smaller number increases (for a review see: Mussolin, Nys, Leybaert, & Content, 2015). However, the extent to which symbolic numbers are rooted in an underlying representation of numerical magnitude shared with nonsymbolic quantities is still an open empirical question (Leibovich & Ansari, 2016). An alternative explanation may be that the overlap in performance profiles is accounted for by shared domain general cognitive resources used for comparing the magnitudes of both nonsymbolic and symbolic numbers. The most likely mechanisms in our opinion are executive function, including inhibitory control, task switching, and working memory, all of which are known to play an important role in math development (e.g. Blair, Knipe, & Gamson, 2008; Blair & Razza, 2007). Therefore, in the current study we examine whether the relation between nonsymbolic and symbolic numerical magnitude comparison is mediated by performance on a mixed format magnitude comparison task, as a measure of shared semantic representation between symbolic and nonsymbolic numbers, and/or by measures of executive function and visuo-spatial working memory. We also examine whether any mediating role of mixed-format comparison performance persists while controlling for performance on executive function and visuo-spatial working memory. According to the mapping hypothesis, if symbolic numbers are grounded in nonsymbolic magnitude representations, then performance on the mixed format comparison task is expected to mediate the relation between them. According to the domain general hypothesis, on the other hand, if the association between numerical formats is not a result of shared underlying representations, then any mediating role of mixed format comparison performance should be accounted for by executive function and/or visuo-spatial working memory.

In summary, the present study addresses two primary questions. First, is the mediating relation between nonsymbolic and symbolic magnitude processing and math mediated by domain general mechanisms of executive function, visuo-spatial working memory, in addition to format specific and cross format numerical comparison? Second, is the relation between nonsymbolic and symbolic magnitude processing driven by a shared underlying representation of magnitude, or by shared domain general cognitive mechanisms?

METHODS

Participants

The current sample was drawn from an ongoing longitudinal study of students who participated in an earlier, short-term longitudinal study of early math skills (Pre-K to 1st grade)(Hofer, Lipsey, Dong, & Farran, 2013). The final analytic sample for the original scale-up study included 771 children. In the follow-up study, we were able to locate 628 students who were attending public school in the 2013-14 year in the same district as they attended in Pre-K (16 had withdrawn from the study in 1st grade and were not contacted for further participation, 29 had moved out of the state, 53 had moved out of the district, and 45 were not located despite all efforts). Of those 628, we were able to obtain parental consent and assess 506 children in the 2014-2015 school year. Our final sample was comprised of 475 students for whom we had complete measures from Pre-K to 6th grade (264 females). Of our 475 students who should have been in 6th grade in the 2014-15 school year if they had not been retained or promoted early, 75 (16%) were still in 5th grade and 1 (0.2%) had been promoted to 7th grade. The sample students were located in 76 schools in the first year of the follow-up study, including 31 elementary schools, 27 middle schools, 11 charter schools, and 7 Innovation Cluster schools (schools that had been targeted for additional resources to boost low student achievement). Family income level was inferred on the basis of whether participants qualified for free or reduced lunches (family income less than 1.85 times the U.S. Federal income poverty guideline). In the current sample 94% of participants qualified for free and reduced lunches.

Table 1. Descriptive statistics for experimental and standardized measures.

N = 475 (267 females)	**Mean**	**SD**	**Range**
Age (years)	12.1	0.3	11.4 –13.4
Nonsymbolic accuracy (%)	75.2	5.3	58.6 – 91.4
Symbolic accuracy (%)	91.2	6.6	63.0 – 100
Mixed Format accuracy (%)	69.9	7.5	53.0 – 93.0
Backward Corsi (max span)	4.92	1.2	2 – 8
Hearts and Flowers accuracy (%)	74.2	1.4	35 – 100
KM-3 Numeration (scale score)	7.9	2.7	2 – 19
KM-3 Algebra (scale score)	8.3	2.9	1 – 17
KM-3 Geometry (scale score)	7.9	2.4	2 – 14
KM-3 Composite (scale score)	8.0	2.4	2 – 16.3

Procedure

All students were consented to participate and the study was approved by the Vanderbilt University IRB. Assessments were conducted by trained members of the research staff of the Peabody Research Institute. The number comparison tasks, cognitive measures, and math achievement measures were administered during the Spring semester of the students' 6^{th} grade year, given that they had not been held back or promoted early. The number comparison tasks and cognitive measures were administered via tablet computers. All testing was completed in a quiet location at the students' school with one-to-one assistance from trained staff.

Number Comparison Tasks

Nonsymbolic Number Comparison. Participants were presented with two sets of dots simultaneously and asked to indicate via button press which set was more numerous (i.e., which set contained more dots). The set on the left side of the screen contained yellow dots and the set on the right side contained blue dots, which corresponded to color-coded left and right buttons. Response side were fully counterbalanced. Trials consisted of 1200 ms stimulus presentation followed by 1800 ms of a fixation cross. Seven ratios were presented, ranging from .33 (5 vs. 15) to .9 (9 vs. 10). The number of dots in each stimulus ranged from 5 to 15. Each ratio was presented 10 times for a total of 70 trials. Ratios, stimulus presentation times, and order of presentation were modeled after Odic, Hock & Halberda (2014). To control for the possibility that participants might choose a strategy based on visual cues rather than number of dots, the following visual properties of dot sets were varied using a modified version of the MATLAB code recommended by Gebuis & Reynvoet (Gebuis & Reynvoet, 2011) to generate stimuli: convex hull (area extended by a stimulus), total surface area (aggregate value of dot surfaces), average dot diameter, and density (convex hull divided by total surface area). In approximately one quarter of the trials all four visual properties were congruent with greater numerosity (i.e. the greater number of dots had a greater convex hull, surface area, etc.). In another approximate quarter of the trials, all four visual properties were incongruent with greater numerosity. In the remaining trials, visual properties were mixed congruent and incongruent.

Symbolic Number Comparison. Participants were simultaneously presented with two, double-digit Arabic numerals and asked to indicate via button press which of the two was numerically larger (e.g., 54 is larger than 18). The ratios presented, order of ratios, and stimuli durations were identical to those in the nonsymbolic number comparison task. To prevent responses uniquely based on the rightmost digit (unit value), the unit–decade compatibility was manipulated such that all trials were decade-incompatible. In other words, the larger number of the pair always had a larger decade but a smaller unit than the smallest number (e.g. 54 vs. 18, 72 vs. 63).

Mixed Format Comparison. Similar to both of the above tasks, participants were presented with two simultaneously presented stimuli, one set of dots and one double-digit Arabic numeral, and asked to indicate via button press which of the two dots was numerically larger (e.g. "12" vs 24 dots). The ratios presented, order of ratios, and stimuli durations were identical to those in the other number comparison tasks. Dot stimuli ranged from 10 to 30 dots per dot set in order to ensure that individuals did not have time to count, given that there was only one set of dots in the mixed-format comparison condition. Both Arabic digits and dot arrays were presented within two grey circles presented on a black background. Arabic digits were presented in black and dots were presented in blue. All of the dots were the same size for the mixed format comparison task.

Number Comparison Task Performance Metrics

A growing body of literature suggests that mean accuracy is highly correlated with and possibly more reliable than ratio dependent metrics such as the *weber fraction* (Gilmore, Attridge, & Inglis, 2011; Inglis & Gilmore, 2014), and that ratio effects are not equivalent across formats (Lyons, Nuerk, & Ansari, 2015). Therefore, in the current study mean accuracy percentages were used to index performance on each of our number comparison tasks.

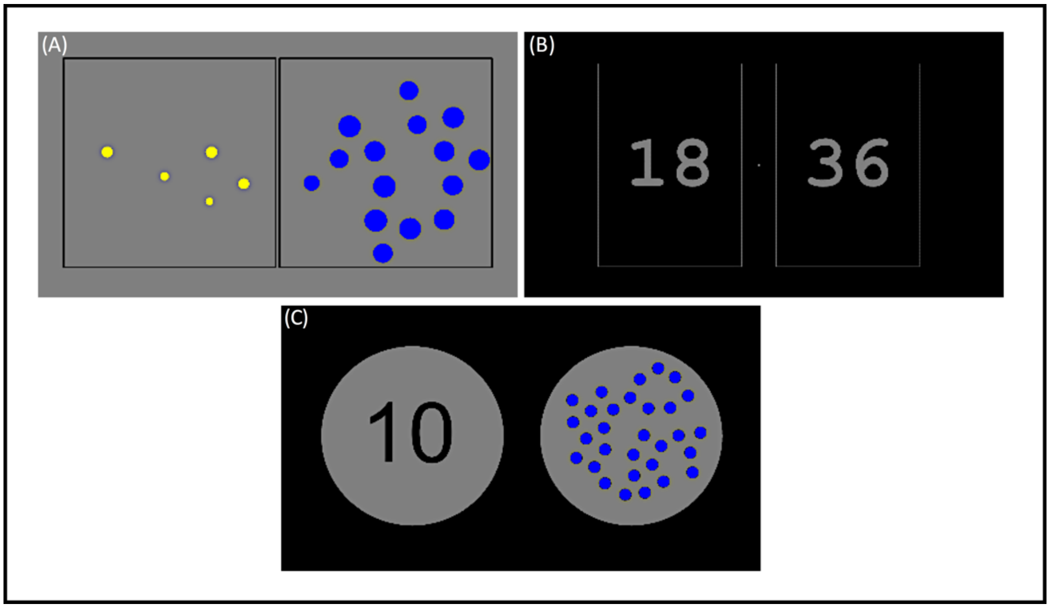

Figure 1. Example screenshots for (A) nonsymbolic, (B) symbolic, and (C) mixed format magnitude comparison tasks.

Cognitive Measures

Working Memory. The backward Corsi block-tapping test (Corsi, 1972) provided a measure of visuo-spatial working memory. In this computerized task, children first viewed squares light up in a sequence on the screen, and then the student were asked to tap the squares in the reverse order from which they lit up. The task consists of 16 total possible trials, including two practice trials. The student was given 2 attempts to correctly repeat the reverse sequence per sequence length. The sequence length of squares increased from 2 to 8 across the activity. If the student correctly answered at least 1 of the 2 attempts correctly, the student then proceeded on to the longer (more difficult) sequence. The score of interest was the highest span with a correctly repeated sequence.

Executive Function. The Hearts and Flowers task (Wright & Diamond, 2014) was used as measure of students' task switching and inhibitory control. In this task, the child was first presented with a heart on either side of the screen and instructed to press the button that corresponds to the side of the screen with the heart. This first block comprised 12 trials. In the second block of trials (also 12 trials), the child was presented with flowers and asked to press the button that is opposite the side of the flower. In the third set of trials, the child was randomly presented with both hearts and flowers and asked to follow the rule that corresponds to hearts and flowers respectively. The third block comprised 48 trials. To index executive function we used mean accuracy from the mixed-condition run, and as such, our measure captures both task switching and inhibitory control. (Diamond, 2014).

Math Achievement Measures

KeyMath 3. The KeyMath 3 Diagnostic Assessment (Connolly, 2007) is a comprehensive, norm-referenced measure of essential mathematical concepts and skills. We used three subscales out of the five subscales in the Basic Concepts area. (1) Numeration: The Numeration subtest measures an individual's understanding of whole and rational numbers. It covers topics such as identifying, representing, comparing, and rounding one-, two-, and three-digit numbers as well as fractions, decimal values, and percentages. It also covers advanced numeration concepts such as exponents, scientific notation, and square roots. (2) Algebra: The Algebra subtest measures an individual's understanding of pre-algebraic and algebraic concepts. It covers topics such as sorting, classifying, and ordering by a variety of attributes; recognizing and describing patterns and functions; working with number sentences, operational properties, variables, expressions, equations, proportions, and functions; and representing mathematical relationships. (3)

Geometry: The Geometry subtest measures an individual's ability to analyze, describe, compare, and classify two-and three-dimensional shapes. It also covers topics such as spatial relationships and reasoning, coordinates, symmetry, and geometric modeling. In order to index a broad range of math achievement, we averaged scale scores from the three subscales into a composite measure (KM Composite). Scale scores in the KeyMath 3 are age-normed to reflect population means of 10 and a standard deviation of 3 for each subtest. Math competence was indexed using a composite score calculated as the mean of the age-scaled scores of the three KeyMath 3 subtests administered, so as to capture performance in a wider range of math skills.

RESULTS

Relations between cognitive and standardized measures

Bivariate correlations between each of the administered measures, as well as the KeyMath 3 composite score are reported in Table 2. To correct for multiple comparisons, the critical p-values for each set of correlations were adjusted using the Benjamini-Hochberg's (B-H) False Discovery Rate method with *Q* (false discovery rate) = .05 (Benjamini & Hochberg, 1995), which provides a good balance between controlling for false positives and power for detecting weaker, but significant relationships. All correlations remained significant after correction.

Table 2. Bivariate correlations of cognitive and standardized test scores.

Measure (N = 472)	1	2	3	4	5	6	7	8	9
1. KeyMath 3 Composite									
2. KeyMath 3 Algebra	.94**								
3. KeyMath 3 Geometry	.87**	.69**							
4. KeyMath 3 Numeration	.94**	.84**	.72**						
5. Nonsymbolic Accuracy	.16**	.13**	.19**	.13**					
6. Symbolic Accuracy	.35**	.35**	.27**	.34**	.23**				
7. Mixed Format Accuracy	.14**	.15**	.10*	.14**	.13**	.25**			
8. Corsi Max Span	.38**	.36**	.32**	.36**	.12**	.22**	.10*		
9. Hearts & Flowers Mixed Block Mean Accuracy	.39**	.37**	.35**	.36**	.21**	.33**	.14**	.21**	

* *p* < .05, ** *p* < .01; Variables 1-4 refer to age-scaled standard scores, variable 8 refers to maximum span achieved.

Mediating relations between nonsymbolic and symbolic magnitude processing and math

To assess the mediating effect of symbolic magnitude processing, executive function, and working memory on the relation between nonsymbolic magnitude processing and math competence, we conducted a simple mediation model using the PROCESS Macro in SPSS (Hayes, 2013). To test for significant indirect effects, we used bootstrapping with 5000 resamples to obtain bias-corrected 95% confidence intervals. If zero is outside the confidence intervals, the indirect effect is consequently not zero and can thus be interpreted as evidence of mediation (Preacher & Hayes, 2008). The confidence intervals resulting from this analysis did not contain zero for any of the mediators (Figure 2), suggesting that symbolic magnitude comparison performance, executive function, and working memory each mediate the relation between nonsymbolic magnitude comparison and math competence.

Recent evidence suggests there may be a bidirectional influence between nonsymbolic and symbolic representations of numerical magnitude, whereby the acquisition of symbolic number knowledge refines the representations in the ANS (Mussolin et al., 2015b; Manuela Piazza et al., 2013). Therefore, we conducted a second mediation analysis in which symbolic magnitude processing was entered as the independent variable predicting KeyMath 3 composite with nonsymbolic magnitude processing as the proposed mediator, in addition to executive function and working memory. The confidence intervals resulting from this analysis did contain zero for nonsymbolic comparison performance, but not for executive function or working memory (Figure 3), indicating that nonsymbolic magnitude comparison performance does not mediate the relation between symbolic magnitude comparison and math competence, while executive function and working memory do.

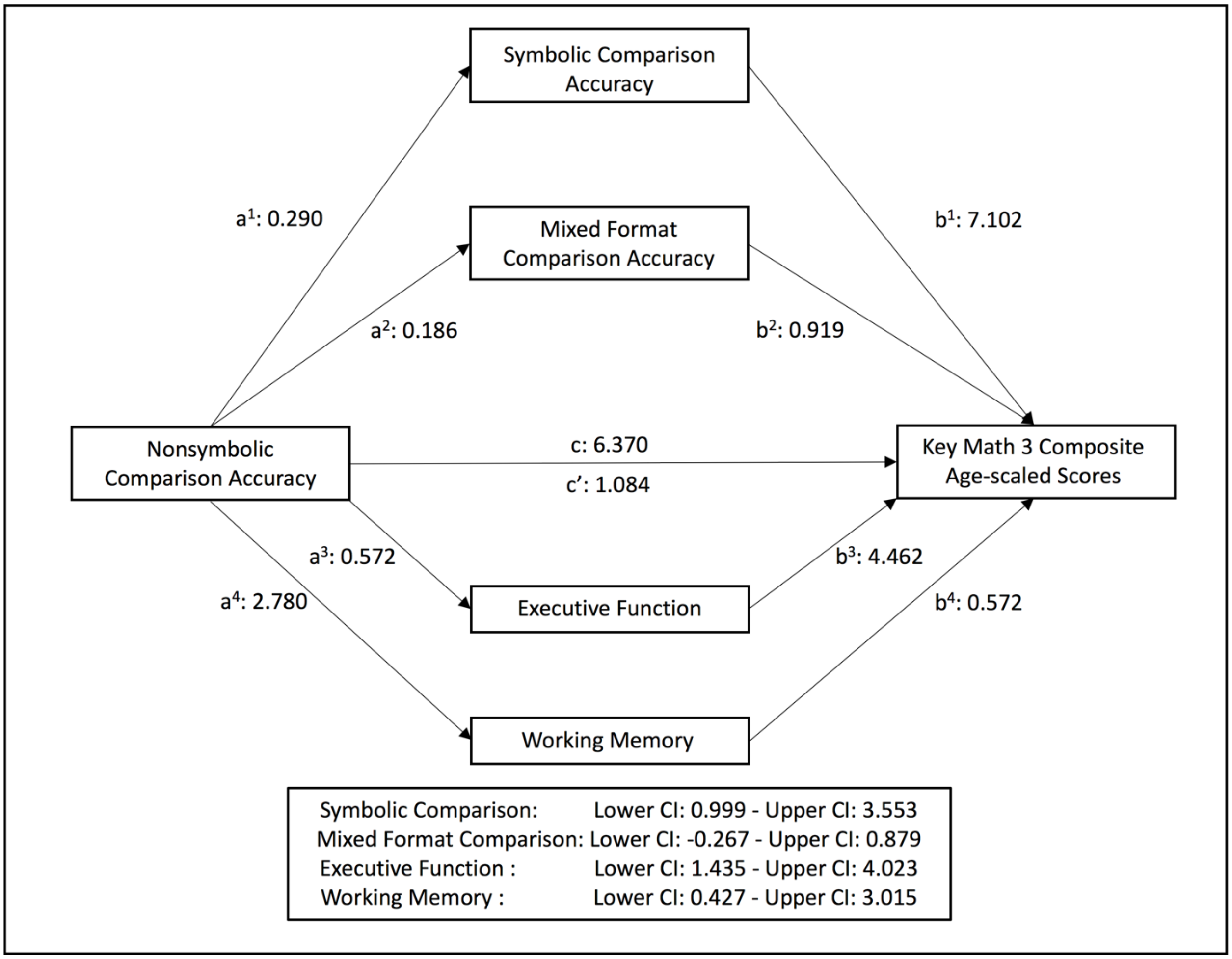

Figure 2. Mediation model showing the relations between nonsymbolic comparison and math performance with symbolic magnitude comparison, mixed format comparison, executive function, and working memory as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

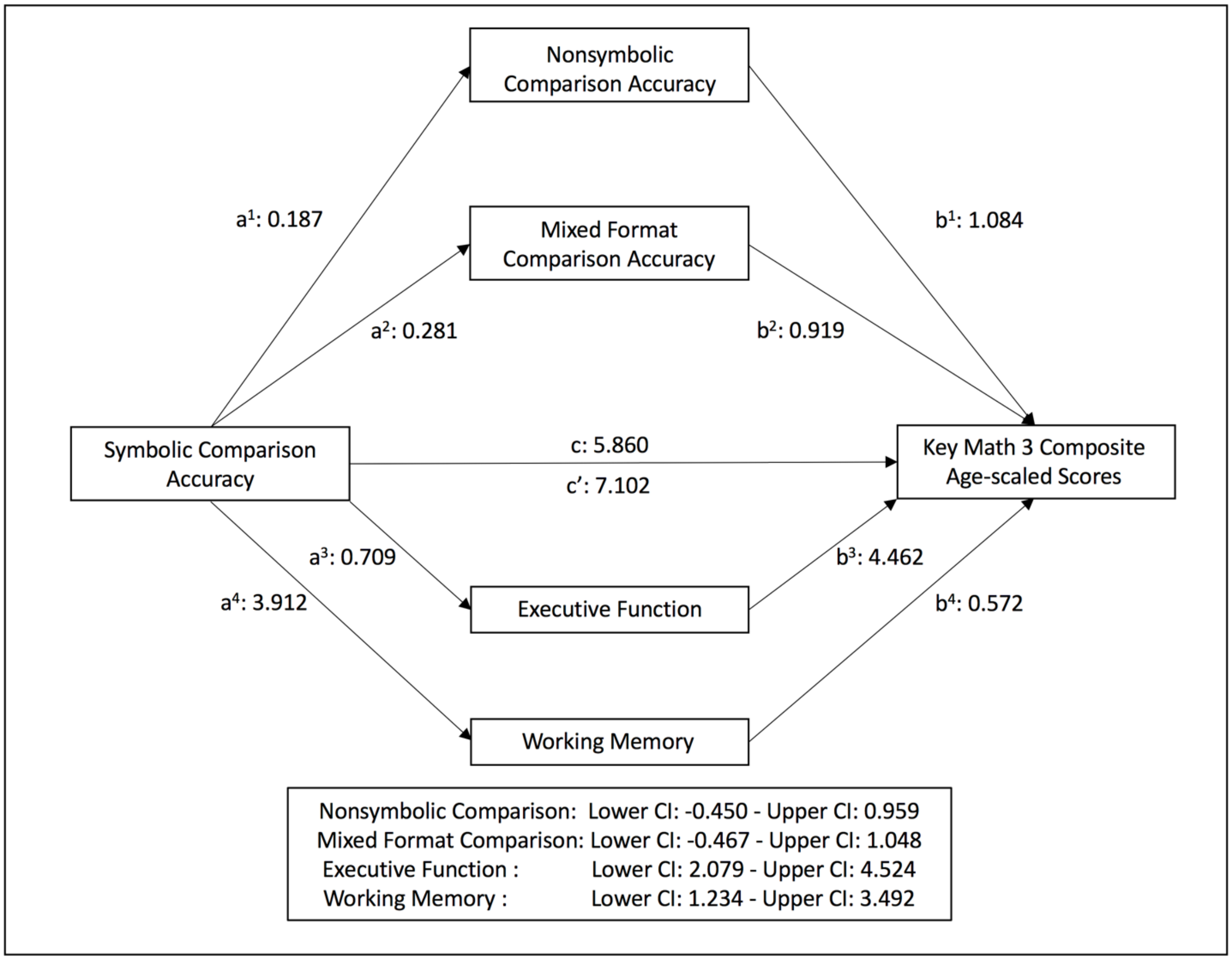

Figure 3. Mediation model showing the relations between symbolic comparison and math performance with nonsymbolic magnitude comparison, mixed format comparison, executive function, and working memory as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

Given that both executive function and working memory mediated the relation between both nonsymbolic and symbolic comparison and math, to assess the extent to which the mediating role of symbolic comparison was influenced by domain general cognitive mechanisms, we replicated model 1 (i.e. symbolic comparison accuracy as the mediator) but included executive function and working memory as covariates instead of mediators. We did not perform this analysis for model 2 because nonsymbolic comparison was not a significant mediator in the original model. The results of this analysis continued to indicate full mediation for symbolic comparison (Lower CI = 0.602; Upper CI = 2.674). These results suggest that the mediating effect of symbolic magnitude processing on the relation between nonsymbolic processing pertains over and above the influence of executive function or working memory.

Finally, to investigate whether the above relations differed as a function of math outcome, we replicated models 1 and 2 using KeyMath Geometry, Algebra, and Number subtests as dependent variables, as opposed to the composite math variable used in our main analyses. The results (Table 3) exactly mirror those when using the composite outcome measure, namely, symbolic comparison mediates the relation between nonsymbolic and math, but nonsymbolic does not mediate the relation between symbolic and math. In other words, there appears to be no difference between sub-tests.

Table 3: Results of the mediation analyses for each KeyMath sub-test as a separate dependent variable.

Math Variable	IV	Mediator	IV to Mediator (a path)	Mediator to DV (b path)	Total Effect of IV on DV (c path)	Direct Effect of IV on DV (c' path)	Lower CI	Upper CI
Geometry	Nonsymbolic	Symbolic	0.290	4.488	4.924	3.607	0.396	2.616
		Mixed	0.186	0.092			-0.491	0.602
		EF	0.572	3.997			1.238	3.729
		WM	2.780	0.476			0.369	2.568
Algebra	Nonsymbolic	Symbolic	0.290	8.719	7.417	-0.123	1.253	4.345
		Mixed	0.186	1.611			-0.198	1.172
		EF	0.572	4.896			1.549	4.501
		WM	2.780	0.644			0.468	3.429
Numeration	Nonsymbolic	Symbolic	0.290	2.346	6.769	-0.232	1.163	4.032
		Mixed	0.186	0.197			-0.317	1.005
		EF	0.572	0.699			1.378	4.140
		WM	2.780	0.695			0.429	3.184
Geometry	Symbolic	Nonsymbolic	0.187	3.607	5.393	4.488	-0.019	1.616
		Mixed	0.281	0.092			-0.728	0.776
		EF	0.709	3.997			1.771	4.147
		WM	3.912	0.476			0.969	3.094
Algebra	Symbolic	Nonsymbolic	0.187	-0.123	6.419	8.719	-0.839	0.802
		Mixed	0.281	1.611			-0.387	1.432
		EF	0.709	4.896			2.212	5.058
		WM	3.912	0.644			1.374	4.012
Numeration	Symbolic	Nonsymbolic	0.187	-0.232	5.769	8.098	-0.788	0.705
		Mixed	0.281	1.055			-0.535	1.238
		EF	0.709	4.492			1.953	4.706
		WM	3.912	0.597			1.268	3.686

Relations between nonsymbolic and symbolic magnitude processing, working memory, executive function, and mixed format magnitude comparison.

To investigate the cognitive mechanisms underlying the relation between nonsymbolic and symbolic magnitude processing, we performed a mediation analysis with nonsymbolic comparison accuracy as the independent variable, symbolic comparison accuracy as the dependent variable, and mixed format comparison accuracy, executive function, and working memory as the proposed mediators. Bias-corrected confidence intervals for the indirect effect in this model did not include zero for any of the mediators, indicating a mediating role for each (Figure 3).

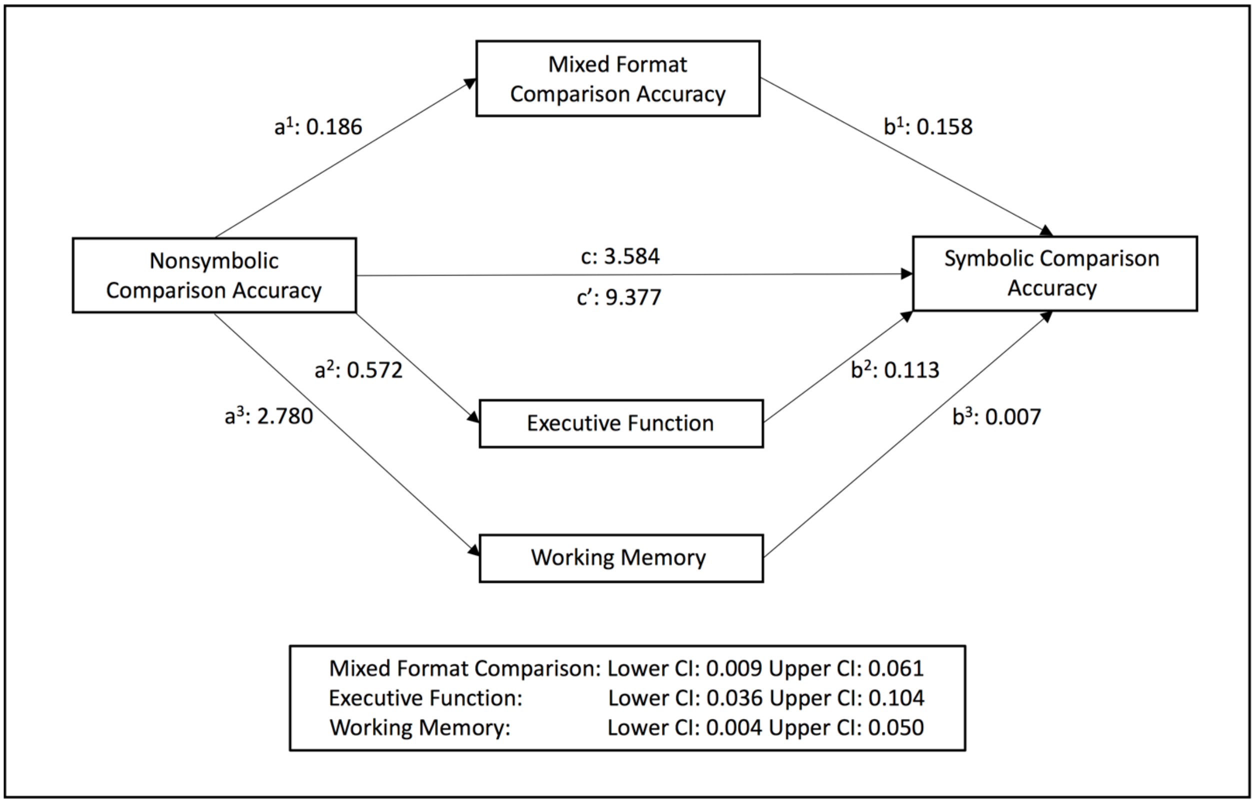

Figure 3. Mediation model showing the relations between nonsymbolic and symbolic magnitude comparison with mixed format comparison, executive function, and visuo-spatial working memory as mediators. Confidence intervals not containing zero are taken to indicate full mediation.

While the executive function and working memory tasks used in this study are widely used indices of those mechanisms, the exact mechanism underlying performance on our mixed format magnitude comparison task is less clear. Although we hypothesize that it indexes some degree of shared semantic representation between nonsymbolic and symbolic number formats, the fact that mixed comparison accuracy correlates with executive function and working memory leaves open the possibility that the mediating role of mixed format comparison reflects the influence of executive function and working memory, as opposed to a distinct mechanism. To test this hypothesis, we performed an additional mediation analyses in which executive function and working memory were entered as covariates in the mediation model between nonsymbolic and symbolic comparison. Bias corrected confidence intervals for the indirect effect did not include zero when controlling for executive function and working memory (Lower CI = 0.003; Upper CI = 0.052) suggesting that mixed format comparison accuracy fully mediates the relation between nonsymbolic and symbolic magnitude comparison independent of the effect of those domain general cognitive mechanisms.

DISCUSSION

Several recent studies suggest that the relation between nonsymbolic magnitude processing skills and math competence is mediated by symbolic number processing skill (Fazio et al., 2014; Lyons & Beilock, 2011; Peng et al., 2017; Price & Fuchs, 2016; Wong et al., 2016). However, less is known about whether mapping between nonsymbolic and symbolic magnitude representations also mediates that relation. It is also unclear whether the mediating role of symbolic number processing is explained by domain-specific number processing, or domain general executive functions. To that end, in the current study we examine whether symbolic comparison, mixed-format comparison, executive function, and visuo-spatial working memory each mediate the relation between nonsymbolic magnitude processing and math.

Our results indicate that symbolic magnitude comparison fully mediates the relation between nonsymbolic magnitude processing and math performance. These findings are consistent with those reported by Price and Fuchs (2016) for typically developing 3rd grade children, and are consistent with an emerging body of literature that suggests symbolic number processing more broadly, not just magnitude comparison, may mediate the influence of nonsymbolic magnitude processing on math development (Fazio et al., 2014; Lyons & Beilock, 2011).

In addition to symbolic magnitude processing, the results of our analyses indicate that executive function and visuo-spatial working memory each mediated the relation between nonsymbolic magnitude processing and math, as well as the relation between symbolic magnitude processing and math. A number of recent studies (Fuhs & McNeil, 2013; Gilmore et al., 2013) suggest that the relation between nonsymbolic magnitude processing and math may be accounted for by executive functions processing related to processing numerical magnitude in the face of conflicting visual cues. The present results support existing findings by showing that both executive function and working memory mediate the relation between nonsymbolic magnitude processing and math, but importantly, symbolic magnitude processing continued to mediate the relation between nonsymbolic magnitude processing and math when controlling for executive function and working memory, suggesting that the influence of symbolic magnitude processing is independent of these domain general cognitive mechanisms. This suggests that both domain specific number processing and domain general cognitive processes are involved in the scaffolding process from basic nonsymbolic magnitude processing to formal math competence. These results were consistent across the geometry, algebra, and number sub-tests of the KeyMath3 battery, suggesting that the observed relations hold true at a broad level and are not unique to specific subdomains of basic math.

Interestingly, our results reveal that mixed-format comparison accuracy did not mediate the relations between nonsymbolic or symbolic comparison and math. If the mixed-format comparison task is taken to index the strength of mapping between nonsymbolic and symbolic representations of numerical magnitude, then these findings are in contrast to those of Wong et al. (2016), who found the number-numerosity mapping, as indexed by dot estimation, mediated the relation between nonsymbolic magnitude processing and math. The most likely explanation for the contradictory results lies in the differences between the 'mapping' tasks. The mixed format comparison task employed in the current study does not require participants to generate a symbolic representation, but rather to compare two simultaneously presented stimuli. It is possible that this process of generating the symbolic output, present in the estimation task employed by Wong et al., engages linguistic or verbal production processes beyond simply transcoding or 'mapping' between the two formats that are pertinent to math development. It is also possible that results of the present study were influenced by the high number of children from low-income backgrounds included in the sample. Individuals from low SES backgrounds typically underachieve in math, with differences already evident in preschool (Sarama & Clements, 2009). A significant body of research suggests that the influence of SES is strongest on verbal and linguistic aspects of mathematics (for a review see Jordan & Levine, 2009), which may alter the influence of nonsymbolic-symbolic mapping processes. However, to fully investigate this possibility, the relation between basic measures of nonsymbolic and symbolic magnitude processing and math in low SES children needs to be empirically examined in contrast to a well-matched control group. Further research is clearly required to understand the source of the differences between the current findings and those of Wong et al..

Our results are also consistent with those reported by Price and Fuchs (2016) and Lyons and Beilock (2011) in that nonsymbolic magnitude processing did not mediate the relation between symbolic magnitude processing and math. Again, these results held true for each of the KeyMath sub-tests. While an emerging body of evidence suggests that the acquisition of symbolic number knowledge may lead to an increase in the precision of nonsymbolic magnitude representations (Mussolin et al., 2015b; Manuela Piazza et al., 2013), the present results indicate a unidirectional influence from nonsymbolic through symbolic to math.

The second principal aim of the current study was to assess whether the relation between nonsymbolic and symbolic magnitude processing is driven by a shared underlying representation of magnitude, or by shared domain general cognitive mechanisms? While much of the extant literature assumes that overlapping performance profiles between nonsymbolic and symbolic magnitude comparison reflect the influence of a shared underlying representation of numerical magnitude, it is also possible that such overlap is the result of shared domain-general executive function mechanisms such as working memory or inhibitory control. The present study investigated this issue by assessing the extent to which the relation between nonsymbolic and symbolic magnitude comparison was mediated by mixed-format magnitude comparison, working memory and executive function. Our results demonstrated that all three measures mediated the relation between nonsymbolic and symbolic comparison, and importantly, that mixed format comparison performance continued to mediate the relation between nonsymbolic and symbolic processing, even when controlling for working memory and executive function. These results suggest that the link between the two number formats is the product of both domain general and domain-specific cognitive mechanisms, and that the mediating role of mixed-format comparison may, at least in part, reflect some degree of shared underlying representation of magnitude between the formats. An alternative explanation is that mixed-format comparison performance reflects the cognitive process of transcoding between numerical formats, and that more efficient transcoding ability enables better learning of numerical symbols. Given the limited literature on mixed-format comparison, and the fact that the current sample included a large proportion of children from low-income backgrounds, these interpretations require further empirical investigation, and importantly, the present results need to be replicated and with children from a full range of income backgrounds.

It should also be noted that, given the amount of assumed cognitive overlap between our three comparison conditions, the strength of statistical associations between them (nonsymbolic-symbolic r = .23, nonsymbolic-mixed r = .13, symbolic-mixed r =.25) were not as strong as might be intuitively expected. However, there are relatively few studies that have utilized the mixed-format experimental paradigm, and results are somewhat mixed. The first study to our knowledge to employ the mixed-format paradigm (Mundy & Gilmore, 2009) reported a lack of significant correlations among tasks in a group of 6- and 8-years-olds. Symbolic comparison correlated with mixed-format comparison at r = -.17, *n.s.*, and nonsymbolic comparison correlated with mixed-format at r = -.04, *n.s.*. However, Brankaer, Ghesquière, & De Smedt (2014) reported significant correlation between accuracy rates for mixed-format and symbolic comparison of r = 0.42, and between mixed-format and nonsymbolic comparison (r = 0.38) in first- and third-graders. Lyons, Ansari, & Beilock (2012) did not report the correlations among these tasks, but do report that mixing symbolic and nonsymbolic representations comes at a significant cost for accuracy, indicating that across format comparisons require additional cognitive resources compared to within-format nonsymbolic or symbolic performance in a group of undergraduates. Therefore, our results fall directly between previously published results, albeit in a different age range, demonstrating a relatively weak but significant relationship. Given the lack of consistency in previous results and the fact that the age of our sample differs from the previous two studies, it remains an open question as to the degree of relation between tasks that require cross-format comparison vs. within-format comparison.

In summary, the present results suggest that the relation between nonsymbolic magnitude processing and math is mediated by both domain-specific symbolic magnitude processing, and by domain general executive function and visuo-spatial working memory. This relationship appears to be unidirectional in that nonsymbolic magnitude processing does not mediate the relation between symbolic comparison and math. In contrast to a recent study, our results suggest that nonsymbolic-symbolic mapping does not mediate the relation between nonsymbolic or symbolic comparison and math. Finally, our results suggest that the relation between nonsymbolic and symbolic magnitude processing is accounted for by executive function, visuo-spatial working memory, as well as nonsymbolic-symbolic mapping. The extent to which performance on the mapping task employed in the current study reflects a shared underlying representation of numerical magnitude versus active transcoding processes, and the extent to which all of the present results generalize to samples from middle and higher income backgrounds requires further empirical investigation.

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Supplementary Table S1. Task details for number comparison tasks of all formats.

	Nonsymbolic	Symbolic	Mixed-format
trials	70 (10 per ratio)	70 (10 per ratio)	70 (10 per ratio)
ratio (numerosities)	0.33 (5 v 15)	0.33 (18 v 54)	0.33 (10 v 30)
	0.5 (5 v 10)	0.5 (36 v 72)	0.5 (12 v 24)
	0.67 (6 v 9)	0.67 (18 v 27)	0.67 (10 v 15)
	0.8 (8 v 10)	0.8 (36 v 45)	0.8 (12 v 15)
	0.86 (12 v 14)	0.86 (54 v 63)	0.86 12 v 14)
	0.88 (7 v 8)	0.88 (63 v 72)	0.88 (14 v 16)
	0.9 (9 v 10)	0.9 (81 v 72)	0.9 (18 v 20)