Early Math Trajectories: From Prekindergarten to Fifth Grade

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ABSTRACT

An Early Math Trajectories model is proposed and evaluated within a longitudinal study of 517 low-income American children from age 4 to 11. In preschool, nonsymbolic quantity, counting and patterning knowledge predicted fifth-grade mathematics achievement. By the end of first grade, symbolic mapping, calculation and patterning knowledge were the key predictors. Further, the first-grade predictors mediated the relation between preschool math knowledge and fifth-grade mathematics achievement.

Background

Mathematics knowledge begins to develop at a young age, and this early knowledge predicts later math and reading achievement (Duncan et al., 2007; Watts et al., 2014).

We propose an Early Math Trajectory Model that encompasses a set of six early math topics that evidence indicates should be of particular importance for supporting mathematics achievement in the middle grades (See Figure)

1. **Nonsymbolic quantity**: Magnitude of sets, without need to use symbols (LeFevre et al., 2010; Libertus et al., 2013).
2. **Counting**: Counting objects, including cardinality (Aunola, et al., 2004).
3. **Symbolic Mapping**: Mapping between symbolic numerals, number names and magnitudes Kolkman et al., 2013; Sasanguie et al., 2012).
4. **Calculation**: Calculating combination or separation of sets (Geary, 2011).
5. **Patterning**: Finding a predictable sequence in repeating patterns (Papic et al., 2011).
6. **Shape**: Identifying shapes and their properties (Clements & Sarama, 2009). Included based on theory rather than evidence.

Current Study

The goal of the current study is to elucidate specific early math knowledge that is predictive of later mathematics achievement for children from low-income backgrounds. We evaluated the Early Math Trajectories model within a longitudinal study of over 500 low-income children from age 4 to 11.

Method

***Participants*:**

•517 students from low-income homes, originally recruited from pre-k classrooms and participating in the Peabody Research Institute Middle School Follow Up Project (56% female; 79% Black, 9% Caucasian).

•Data collected at beginning and end of pre-k (*M* = 4.4 and 5.0, respectively), end of first grade (*M* = 7.0) and end of fifth-grade (*M* = 11.0) though 14% had been retained in 4th grade

Early Predictor Measures:

•Research-based Early Maths Assessment (REMA; Clements, Sarama & Liu, 2008). Numeracy items were broken into four subscales, in line with past research (Purpura & Lonigan, 2013), and patterning and shape items were treated as separate subscales.

•General cognitive and academic skills were assessed, including *narrative recall* skills (using the Renfrew Bus Story or Woodcock Johnson III Story recall, depending on time point) and *reading* skill (using WJ Letter-Word Identification), and teacher ratings of *attentive behavior* (Cooper-Farran work-related skills) and *self-regulation* (from Instrumental Competence Scale for Young Children-Short Form).

Age 11 Mathematics Achievement Measures:

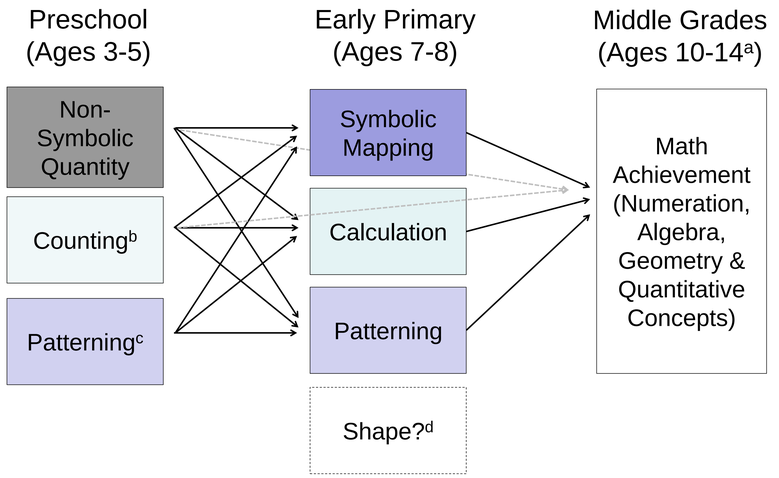
•Composite math achievement measure: Sum z-scores on 3 KeyMath 3 Diagnostic Assessment subtests - Numeration, Algebra and Geometry - and WJ III Quantitative Concept subtest (individually administered)

Table 1: Regression estimates predicting 5th grade math from early knowledge

Measure	Begin of pre-k *M* Age = 4.4	End of pre-k *M* Age = 5.0	End of first grade *M* Age = 7.0
Math Predictors			
Counting	.13*	-.02	-.01
Nonsymbolic	.13**	.19***	.03
Symbolic Mapping	.06	.11	.26***
Shape	.05	.03	.03
Patterning	--	.18***	.08*
Calculation	--	--	.24***
Narrative Recall	.11*	.17***	.09**
Reading	.04	.10*	.11**
Work-related Skills	.10	.08	.18**
Self-Regulation	.05	.01	.01
Controls	Inc.	Inc.	Inc.

Note: All variables were standardized and standardized regression coefficients are reported. Control variables included ratings of attentive behavior and self-regulation, gender, ethnicity, SES composite with maternal education and level of income, ELL status, PreK school type, age at time of testing at both time points & grade level at Age 11, *p < .05. **p < .01. ***p < .001.

Early Math Trajectories Model



Note. [a]Current evidence is for ages 10-11 [b]A reliable predictor at beginning of pre-k, but not the end of pre-k. [c]Not adequately assessed at beginning of pre-k to include in analyses [d]Not a unique predictor.

Sample Math Items

Math Topic	Easier Item	Harder Item
Counting	Shown 5 objects in a line: "'I bought these cans of food. Count these cans to tell me how many there are.'	Count 30 pennies and identify how many.
Nonsymbolic Quantity	Shown two cards, with 4 dots and 3 dots: 'Which one has more?'	Put connected cube towers in order from smallest to largest (towers made of 6-12 cubes)
Symbolic Mapping	Match the numerals 1-5 to the appropriate number of grapes.	Asked: Which is smaller, 27 or 32?
Shape	Select all triangles from a collection of 24 shapes; some are prototypic shapes and some are not.	Fill 6 outlines of regular hexagons with pattern blocks, using different compositions for each
Patterning	Identify the missing element in the pattern ABA_AB	Duplicate a single copy of the core unit of a pattern
Calculation	"Here are 6 pennies. Three more are hidden under the cloth. How many are there in all?"	Add 3 to 69

Table 2: Mediation Model

Beginning of pre-k predictors	Without mediators	With mediators
Counting	.23***	.09*
Nonsymbolic Quant.	.16***	.01
Narrative Recall	.13**	.06
End of first-grade mediators		
Symbolic Mapping	--	.36***
Patterning	--	.10**
Calculation	--	.29***
Controls	Inc.	Inc.

Results

In preschool, individual differences in nonsymbolic quantity, counting and patterning knowledge predicted fifth-grade mathematics achievement over and above many other math and cognitive skills (see Table 1).

In first grade, individual differences in symbolic mapping, calculation and patterning knowledge were key predictors of later math achievement; shape knowledge was not (see Table 1).

First-grade knowledge mediated the relation between preschool math knowledge and fifth-grade mathematics achievement. See Table 2 for beginning of pre-k models. Symbolic mapping and calculation knowledge were significantly stronger mediators than patterning, using a bootstrapping technique.

Conclusion

Strong support for Early Math Trajectories model for low-income children.

Early patterning knowledge merits increased attention in theories of math development and should be included in early standards (contrary to Common Core State Standards, 2010).

Nonsymbolic quantity knowledge in preschool may have an indirect effect on later achievement (see DeSmedt et al., 2013). It may support several primary-grade math topics, including symbolic mapping, calculation and patterning knowledge.